QUANTIZATION OF CHIRAL SOLITONS FOR THREE FLAVORS AND THE LARGE-N LIMIT

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The quantization of a skyrmion in a theory with three flavors is discussed. The conventional choice of octets does not maintain correspondence with the large-N limit of the quark model. When the skyrmion is quantized so as to maintain this correspondence in the spectrum of states, magnetic moments computed in the skyrmion model and the large-N quark model also agree (in the limit of flavor SU3 symmetry). When flavor SU3 is broken sum rules are found in the quark model valid for any number of colors which connect magnetic moments of spin 1/2 baryons.

Quantum chromodynamics is the theory underlying hadron physics. However we do not yet know how to solve this theory, so that we need approximations. One way of approximating the theory is to consider the limit as the number of colors N increases to infinity [1,2]. Witten has shown that in this limit baryons behave as solitons in an effective meson field theory. Balachandran, Witten and others [3,4] found that the precise connection between baryons and mesons in this limit is the one discovered by Skyrme many years ago.

In particular, with quarks of two flavors only, Witten has shown that the spectrum of the quantized skyrmion

$$I = J = 1/2, 3/2, 5/2, \ldots \tag{1}$$

is the same as the spectrum of a multiquark baryon of large (odd) N. We shall call this multiquark model the naive quark model of arbitrary color. The states of this system can be derived from the assumption that its color state is completely antisymmetric under permutations. It has also been noted that several matrix elements of simple operators computed in the Skyrme model have identical ratios in the naive quark model of arbitrary color as N becomes large [5,6]. This suggests that the Skyrme model is a good model of the baryon in the large-N limit of QCD. Of course in nature $N = 3$ and only multiplets with $I = J = 1/2, 3/2$ are physical.

We are interested here in the same correspondence when the quarks are allowed to have three flavors. The quantum numbers allowed for the quantized skyrmion are more difficult to determine, since as noted by Witten the spectrum depends on the choice of the Wess–Zumino term in the lagrangian. Witten, Guadagnini and others [7,8] have shown that one can quantize the skyrmion so that the multiplets have the

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content under SU3 × SU2 of (8, 1/2) and (10, 3/2), in agreement with experiment. The skyrmion model quantized in this fashion has been used to compute matrix elements for various operators relevant to weak and electromagnetic transitions in hyperons [9,10].

We compute here the SU3 × SU2 spectrum expected for three flavors in the naive quark model for arbitrary color and find that the physical multiplets (8, 1/2) and (10, 3/2) do not appear at all for N other than three. Thus for three flavors the skyrmion quantized so as to give octets and decuplets should not be thought of as a good model for baryons in the large-N limit of QCD, in contrast to the case of two flavors.

We use the notation (p, q) for an irreducible representation of SU3. In this notation a totally symmetric representation of SU3 composed of n quarks is denoted by $(n, 0)$. For example the decuplet is (3, 0). To find the SU3 content of an arbitrary baryon composed of $(2k + 1)$ quarks, it is sufficient to consider the states which contain $(k + 1)$ up quarks and k down quarks. The up quarks will belong to the $((k + 1), 0)$ representation of SU3, while the down quarks belong to the $(k, 0)$ representation. Therefore we have to take the direct product of these two representations of SU3, and of the corresponding spins: $S_u = (k + 1)/2$ and $S_d = k/2$.

We use the SU3 formula

$$(k+1,0)\times(k,0)=(2k+1,0)+(2k-1,1)+\ldots+(1,k)\,, \qquad (2)$$

where the spin of each multiplet (i, j) is $i/2$. For example, for $k = 1$ $(N = 3)$ eq. (3) reduces to the familiar result

$$(2,0)\times(1,0)=(3,0)+(1,1)\,, \qquad (3)$$

a decuplet of spin 3/2 and an octet of spin 1/2. Therefore in contrast to the two-flavor case the SU3 representation content for N colors is completely different at each value of N: a given representation (i, j) of SU3 occurs at most once at $N = 2j + i$. In particular the octet (1, 1) only occurs for $N = 3$, while for larger N values the spin 1/2 states belong to successively larger representations of SU3. With three quark flavors and more than three colors not only are the multiplets with spin 5/2, 7/2, ... unphysical, but even in the multiplets with physical spin there are states with unphysical quantum numbers in addition to the states with physical quantum numbers. The physical states have at most two (at spin 1/2) or three (at spin 3/2) strange quarks while the unphysical states have a larger number of strange quarks. Even though for large N the multiplets have very many unphysical states we may still compute matrix elements relevant to the physical states, and it is the limit of such matrix elements for large N that is of theoretical interest.

This discussion then suggests that we quantize the skyrmion in the same $(1, k)$ representation of SU3 and take the limit of matrix elements as k becomes large, to obtain results relevant to baryons of spin 1/2. We discuss here matrix elements of the magnetic-moment operator as a simple example of such a computation. The formalism we follow has been described in the literature by Adkins and Nappi and by Wise et al. [9,10], for the special case $k = 1$ when the magnetic moment of a baryon, say, the proton, is given by the expression

$$\langle \mathrm{P}|\mu_z|\mathrm{P}\rangle=\zeta\sum\begin{pmatrix}8 & 8 & n\\ Q & \mathrm{P} & \mathrm{P}\end{pmatrix}\begin{pmatrix}8 & 8 & n\\ \pi^0 & \mathrm{N} & \mathrm{N}\end{pmatrix}, \qquad (4)$$

where Q is the linear combination $(\pi^0 + (1/\sqrt{3})\eta)$ corresponding to the transformation properties of the electric-charge operator in SU3, the second CG coefficient accounts for the coupling of the spin of the states to the magnetic-moment operator and the intermediate multiplets n correspond to two octets. In the case of arbitrary N the charge Q still transforms like an element of an octet but the second and third multiplets (in both CG coefficients) are replaced by $(1, k)$, and just as in the case of octets the representation $(1, k)$ occurs twice in the product $(1, 1) \times (1, k)$.

We have computed the CG coefficients relevant to the evaluation of the magnetic moment of the proton, neutron and other baryons of spin 1/2, some of which are given below. Because the representation $(1, k)$ occurs twice there is an arbitrariness in the choice of multiplets n which we have taken advantage of to simplify the algebraic expressions of the CG coefficients. In the case $k = 1$ our choices do *not* reduce to the well-known 8a, 8s basis traditional for the product of two octets in the literature. However, the final answers for physical quantities like magnetic moments does not depend on arbitrary choices, as may be seen by computing the ratio of the magnetic moment of the proton to the magnetic moment of the neutron.

We give below some CG coefficients for the multiplication of $(1, k)$ by $(1, 1)$; the states in the octet $(1, 1)$ are denoted by meson labels while the elements of $(1, k)$ are denoted by baryons with the appropriate quantum numbers. We denote the two $(1, k)$ multiplets by A and B in these formulae,

$$\begin{pmatrix} 8 & (1,k) & (1,k)\mathrm{A} \\ \pi^0 & \mathrm{P} & \mathrm{P} \end{pmatrix} = [k/6(k+2)]^{1/2} ,$$

$$\begin{pmatrix} 8 & (1,k) & (1,k)\mathrm{A} \\ \eta & \mathrm{P} & \mathrm{P} \end{pmatrix} = -[k/2(k+2)]^{1/2} ,$$

$$\begin{pmatrix} 8 & (1,k) & (1,k)\mathrm{B} \\ \pi^0 & \mathrm{P} & \mathrm{P} \end{pmatrix} = (k+3)/[6(k+2)(k+4)]^{1/2} ,$$

$$\begin{pmatrix} 8 & (1,k) & (1,k)\mathrm{B} \\ \eta & \mathrm{P} & \mathrm{P} \end{pmatrix} = (k+1)/[2(k+2)(k+4)]^{1/2}, \quad (5)$$

in particular, for $k = 1$, we have the values $1/\sqrt{18}$, $-1/\sqrt{6}$, $4/\sqrt{90}$, $2/\sqrt{30}$. It can be checked that these values are linear combinations of CG coefficients for the more usual 8a and 8s in the literature. Using these values we can evaluate formula (7) with the intermediate multiplets 8_A and 8_B and find for the magnetic moments of the proton and neutron

$$\mu(\mathrm{P}) = -4\zeta/15 , \quad \text{and} \quad \mu(\mathrm{N}) = 3\zeta/15 , \quad (6)$$

and therefore the ratio

$$\mu(\mathrm{P})/\mu(\mathrm{N}) = -4/3 , \quad (7)$$

in agreement with the result in the literature [9,10] corresponding to the quantization of the skyrmion as an octet. On the other hand we can take the limit of the CG coefficients as k becomes very large and then compute from the formula (7) the values

$$\mu(\mathrm{P}) = -\zeta/3 \quad \text{and} \quad \mu(\mathrm{N}) = \zeta/3 , \quad (8)$$

and therefore the ratio $\mu(\mathrm{P})/\mu(\mathrm{N}) = -1$. This ratio is in agreement with the result obtained for the naive quark model in the limit of large numbers of colors [5]. The agreement between these two results suggests that the ratio of minus one is characteristic for the large-N limit of QCD.

In the same limit of k becoming large we can compute the ratio of other magnetic moments, and again the results of the Skyrme model and the naive quark model agree. These results are relevant to the flavor SU3 symmetry limit when the strange quark and the down quark are degenerate and have the same magnetic moments. In this limit we find

$$\mu(\Sigma^+)/\mu(\mathrm{P}) = 1 , \quad \mu(\Sigma^-)/\mu(\mathrm{P}) = -1 ,$$
$$\mu(\Lambda)/\mu(\mathrm{P}) = 0 , \quad \mu(\Xi^-)/\mu(\mathrm{P}) = 1/3 ,$$
$$\mu(\Sigma\Lambda)/\mu(\mathrm{P}) = 1 , \quad \mu(\Xi^0)/\mu(\mathrm{P}) = -1/3 . \quad (9)$$

When flavor SU3 is broken, the down quark and the strange quark have different magnetic moments and in the naive quark model or arbitrary color the magnetic moment of a baryon is a function of k and of the magnetic moments of the up, down and strange quark. With seven "stable" baryons of spin one half we can eliminate four parameters and obtain three sum rules which are valid for arbitrary color group $\mathrm{SU}(2k + 1)$ and arbitrary breaking of flavor SU3. We find the following sum rules:

$$4[\mu(\mathrm{P}) + \mu(\mathrm{N})] + 2\mu(\Lambda) = 3[\mu(\Sigma^+) + \mu(\Sigma^-)] , \quad (10)$$

$$-[\mu(\mathrm{P}) + \mu(\mathrm{N})] + 8\mu(\Lambda) = 3[\mu(\Xi^0) + \mu(\Xi^-)] , \quad (11)$$

$$[\mu(\mathrm{P}) - \mu(\mathrm{N})] - 2[\mu(\Sigma^+) - \mu(\Sigma^-)] = 3[\mu(\Xi^0) - \mu(\Xi^-)] . \quad (12)$$

These sum rules are satisfied for the magnetic moments (9), and also by the usual quark model or SU6 formulae; as far as we know the sum rules are new since there is no particular motivation to take these linear combinations when there are only three colors. The last sum rule relates the isovector components of the magnetic moments of baryons with different numbers of strange quarks and is fairly badly violated by the experimental values, by about 25%. This might be due to the neglect of isovector contributions coming from pions, etc. which are not considered in these formulae. The first two sum rules relate isoscalar components which should not be affected by our neglect of pions. The first sum rule is not well obeyed by the data (error of 20%) while the second sum rule is very well obeyed by the data (error of 1%). It is tempting to conclude that there is some experimental error in the isoscalar component of the magnetic moment of the Σ baryons but it may be that the agreement of eq. (11) with the data is fortuitous. More details on these issues will be presented elsewhere.

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References

[1] G. 't Hooft, Nucl. Phys. B 72 (1974) 461.
[2] E. Witten, Nucl. Phys. B 160 (1979) 57.
[3] A.P. Balachandran, V.P. Nair and S.G. Rajeev, Phys. Rev. Lett. 49 (1982) 1124.
[4] G.S. Adkins, C.R. Nappi and E. Witten, Nucl. Phys. B 228 (1983) 552.
[5] G. Karl and J.E. Paton, Phys. Rev. D 30 (1984) 238.
[6] A.V. Manohar, Nucl. Phys. B 248 (1984) 19.
[7] E. Witten, Nucl. Phys. B 223 (1983) 422.
[8] E. Guadagnini, Nucl. Phys. B 236 (1983) 35.
[9] J. Bijnens, H. Sonoda and M.B. Wise, Phys. Lett. B 140 (1984) 421.
[10] G.S. Adkins and C.R. Nappi, Nucl. Phys. B 249 (1985) 507.