The theory of tax evasion: A retrospective view.

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Abstract.

The paper gives an overview of some main themes in the theory of tax evasion, starting from Allingham and Sandmo (1972). It reviews the comparative statics of the original model of individual behaviour where the tax evasion decision is analogous to portfolio choice, and its extensions to incorporate socially conscious behaviour, participation in the black labour market and tax evasion by firms. It also discusses the analysis of tax incidence and the problems involved in moving from individual to aggregate analysis. Finally, it reviews the issues that arise in formulating models of optimal taxation in the presence of tax evasion.

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1. Introduction.

If someone were to write a full history of taxation, including both practitioners’ experience and the thinking of theorists, it is probably a good guess that tax evasion would be part of the picture from the very start. The formal economic theory of tax evasion, on the other hand, is of considerably more recent origin and started to develop only a little over thirty years ago. To the best of my knowledge, its beginning can be dated to 1972\(^1\) with the publication of the article “Income tax evasion: A theoretical analysis” by Michael Allingham and myself (Allingham and Sandmo 1972)\(^2\). It was followed by a large number of contributions to the literature which extended the original model in a number of directions. The present paper, although not attempting a complete survey of the literature, reviews the main problems and developments in the theoretical literature on tax evasion and relates it to other issues that have traditionally been central in the theory of public finance.

Recent decades have also seen a number of attempts to provide empirical estimates of the size of the “hidden economy”. Although little of the empirical research, at least to begin with, was based on an underlying theoretical structure, there is no doubt that the empirical work and the policy discussions that followed from it gave inspiration to further theoretical work, and that theory also gave new directions for empirical investigations. The literature through the 1980s was very nicely surveyed by Cowell (1990); more recent surveys include Andreoni, Erard and Feinstein (1998), Slemrod and Yitzhaki (2002) and Cowell (2002).

In the present paper I wish to take my point of departure from the Allingham-Sandmo (A-S) article, describe its basic structure and consider some of its weaknesses in the light of more recent developments. I then go on to discuss more normative issues like the implications of

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\(^1\) The paper by Srinivasan (1973), which was written at about the same time, assumes that the taxpayer is risk neutral, maximizing expected after-tax income, which is a special case of the A-S analysis. On the other hand, he allows both the regular and the penalty tax schedules to be progressive, which is more general than the A-S formulation.

\(^2\) The work on the paper started in the summer of 1971, when the two of us met in Bergen for an extended (four weeks!) summer workshop, organized by the International Economic Association. The purpose of the workshop was to bring together a number of young European economists working in areas which at the time were at the forefront of interest among theoretical economists. Two of these areas were public economics and the economics of uncertainty, and for those with an interest in both fields, the economics of tax evasion was a perfect topic.
the tax evasion literature for the theory of optimal tax design and the analysis of tax administration. But before going into these more analytical parts of the paper, it will be useful to start with some more general perspectives on this part of the theory of taxation.

2. The costs of tax administration.

It is an old insight, going back at least to Adam Smith (1776, Book V, Chapter II) that one of the demands that we should make on a good tax system is that the costs of administration are low. In the modern theory of taxation the costs of the tax system have mostly been associated with the efficiency costs of tax wedges that arise because of the distortions of the competitive price mechanism. The more direct costs of tax collection have, by contrast, practically been neglected. This is unfortunate both from a positive and normative point of view. On the one hand, tax collection involve costs incurred by the public tax authorities in assessing tax liabilities, reviewing tax returns, pursuing evaders etc. On the other hand, there are the costs that taxpayers carry by spending time to read up on the tax rules and filling out their tax return forms. In the case of firms, a major cost to them is doing a significant part of the work of actually collecting the taxes for the government, both in the case of indirect taxes, payroll taxes and income taxes. If the private costs of tax compliance vary between branches of industry, modes of business organization and personal occupation, there is every reason to believe that people’s choices will be affected by these differences in cost. Someone who is about to decide whether to set up his own business or accept a salaried position in a big company, may let his choice be influenced by the consideration that in the former case his costs of tax compliance - as well as the opportunities for tax evasion and avoidance - are likely to be much higher. So the costs of tax compliance on the part of the taxpayer, which form part of the economy-wide costs of the tax system, are likely to have effects on the structure of industry and occupations in a country, and in the next round on returns to investment and gross wages. This has so far been a rather neglected area of public economics, at least in the theoretical literature.

The theory of optimal taxation can be seen as a recipe for minimizing the costs of taxation. The costs on which this literature focuses are, as already noted, the efficiency costs of a distorted tax system. But the more direct costs of administration and compliance play little or

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3 In the final instance, of course, all costs of administration must be borne by the consumers.
no role in the analyses, and the theories of tax evasion that will be discussed below alert us to some of the important aspects of these costs. So far, the potential gains from using the insights of the tax evasion literature in the study of optimal taxation have not been fully exploited, although for some aspects of taxation the evasion perspective is obviously highly relevant. This is true e.g. with respect to the degree of progressivity of the personal income tax, the interface between personal and company taxation and the degree of differentiation of the indirect tax system. The literature on tax evasion should be seen as a way to bring issues of tax administration into the focus of the theoretical literature on tax design.

3. Evasion and avoidance.

The conceptual distinction between tax evasion and tax avoidance hinges on the legality of the taxpayer’s actions. Tax evasion is a violation of the law: When the taxpayer refrains from reporting income from labour or capital which is in principle taxable, he engages in an illegal activity that makes him liable to administrative or legal action from the authorities. In evading taxes, he worries about the possibility of his actions being detected. Tax avoidance, on the other hand, is within the legal framework of the tax law. It consists in exploiting loopholes in the tax law in order to reduce one’s tax liability; converting labour income into capital income that is taxed at a lower rate provides one class of examples of tax avoidance. In engaging in tax avoidance, the taxpayer has no reason to worry about possible detection; quite the contrary, it is often imperative that he makes a detailed statement about his transactions in order to ensure that he gets the tax reduction that he desires.

If tax avoidance is legal, what is the difference between avoidance and the reaction to high taxes that arises because of price effects on demand and supply? Suppose a higher tax on air travel makes me travel more by train, or that a higher marginal tax on labour income makes me switch some hours from work into leisure activities. Am I then engaging in tax avoidance? A simplistic definition of tax avoidance is one that focuses on the lawmakers’ intention and says that avoidance is a type of action that is an unintended although legal consequence of tax policy. By this definition the price effects should perhaps not be classified as avoidance. However, it is often far from simple to discover what the intentions of politicians really are. Official estimates of the revenue effects of tax changes often assume that tax bases are constant, which suggests that political intentions are formed on the assumption that price elasticities are zero. But when a tax increase leads to a reduction of the quantity demanded
and supplied and therefore to lower revenue than the official estimate, one could then classify this as an unintended effect of the tax increase, so that the price effect becomes a kind of avoidance. Clearly, the simplistic definition fails to capture the distinction between tax avoidance as a specific type of activity and effects on demand and supply via relative price effects. Slemrod and Yitzhaki (2002) argue that avoidance consists in actions that do not change the individual’s consumption basket (which presumably includes his consumption of leisure), and that this distinguishes it from real substitution responses. This definition focuses on the absence of relative price changes for consumption goods, but it neglects the income effects that arise from increases in disposable income. Perhaps the borderline between tax avoidance and “ordinary” demand and supply effects must by necessity remain somewhat vague.

There would not be much reason to worry about these distinctions, were it not for the fact that many people have difficulties in seeing the difference between tax evasion and avoidance from a moral point of view. The house painter who does a bit of extra work in the black economy violates the law, while the wealthy investor who engages a tax lawyer to look for tax havens does not. However, from a moral point of view their ways of behaviour may not seem to be all that different. Clearly, the borderline between what seems morally right and wrong does not always coincide with the border between what is legal and illegal. This should be kept in mind when considering the theoretical literature on tax evasion, where the basic assumption is that the taxpayer wishes to hide his actions from the tax collector.

4. The A-S model.

The structure of the Allingham-Sandmo (A-S) model is very simple. No account is made of the taxpayer’s “real” decisions; his labour supply and therefore his gross earnings are taken as given, and the same is true of his income from capital. The model pictures the taxpayer at the moment of filling in his income tax return: How much of his income should he report and how much should he evade?

Let $W$ be the gross income of the taxpayer. There is a proportional income tax at the rate $t$. The amount evaded, i.e. the amount of underreporting, is $E$, so that the reported income is
If the tax evasion is not detected by the tax authority, the net income of the taxpayer is accordingly

\[ Y = W - t(W - E) = (1 - t)W + tE. \]  
(1)

If, however, it is discovered that the taxpayer has underreported his income, he will pay a penalty rate\(^5\) of tax, \(\gamma\), on the evaded amount, so that his net income in this case is

\[ Z = (1 - t)W + tE - \gamma E = (1 - t)W - \gamma tE. \]  
(2)

It should be pointed out that one obviously unrealistic simplification in this model is the assumption that all income is equally unknown to the tax collector. This is clearly not the case; in most countries earnings are reported to the tax authorities by the employer, so that this part of his income cannot in fact be underreported by the employee - unless he acts in collusion with his employer. The analysis should therefore be interpreted as applying to that part of his income which the taxpayer can in fact evade without certainty of detection.

The taxpayer’s subjective probability of detection is \(p\). He chooses the amount evaded so as to maximize his expected utility, which is

\[ V = (1 - p)U(Y) + pU(Z). \]  
(3)

It is assumed that \(U\) is increasing and concave, so that the taxpayer is risk averse. The first-order condition for an interior solution is

\[ (1 - p)U'(Y) t - pU'(Z)(\gamma - t) = 0, \]  
(4)

or

\[ U'(Z)/U'(Y) = (1 - p)t/p(\gamma - t). \]  
(5)

amount of income evaded, rather than reported, as the decision variable of the taxpayer; this seems a more natural formulation and leads to more unambiguous results.

\(^5\) This terminology has become common usage, although it does not correspond to the more everyday meaning of "penalty rate", which should rather be identified with \(\gamma t\).
To see the empirical implications of the model one has to differentiate the first order conditions with respect to the exogenous variables $W$, $t$, $?$, and $p$. It turns out that the signs of the derivatives $?E/??$ and $?E/?p$ are both unambiguously negative; a higher penalty rate or a higher probability of detection always tend to discourage tax evasion. Intuitively, this is seen from (5) by noting that the right-hand side of the equation can be interpreted as the relative price of income in the states of detection and non-detection, and this depends negatively on $?$ and $p$. When $?$ or $p$ increases, $Z$ increases relative to $Y$, which implies that there must be less evasion.\(^6\)

It seems reasonable to assume that a higher gross income will increase evasion if one believes that people become more willing to engage in risky activities as they get richer. This is also predicted by the model if one makes the additional and common assumption that the measure of absolute risk aversion (defined as $-U''(\cdot)/U'(\cdot)$) is decreasing. As regards the effect of the regular marginal tax rate, a notable feature of the original A-S model is that an increase of the tax rate has an ambiguous effect on tax evasion. There is an income effect which is negative; higher taxes make the taxpayer poorer and therefore less willing to take risks. But there is also a substitution effect that works in the direction of increased evasion. In fact, the effect of the marginal tax rate on evasion can be written as

$$\frac{?E}{?t} = -\left(\frac{W-E}{1-t}\right)(?E/?W) + S.$$ \hspace{1cm} (6)

Here $S$ is the substitution effect, which is positive. The first term on the right is the income effect, and this is negative if evasion increases with gross income. The income and tax effects are derived in the Appendix to the present paper.

Yitzhaki (1974) pointed out that this result depends crucially on the assumption that the penalty is imposed on the amount of income evaded. If instead the fine is imposed on the evaded tax, as in the cases of the American and Israeli tax laws, there would be no substitution effect and accordingly no ambiguity. This is easily seen if we redefine the penalty rate to apply to the evaded tax, so that the penalty to be paid is $?tE$ with $?t > 1$. Equation (2) then becomes

\(^6\text{Christiansen (1980) shows that if the expected gain from tax evasion is held constant, an increase of the penalty rate combined with a decrease of the probability of detection will always reduce tax evasion.}\)
\[ Z = (1-t)W-(\rho-1)tE. \]  

(2a)

The first order condition (5) must then be rewritten as

\[ \frac{U'(Z)}{U'(Y)} = \frac{(1-p)/p}{(\rho-1)}, \]  

(5a)

so that the relative price of income in the two states is now independent of \( t \). There remains only the income effect, which establishes a negative relationship between the tax rate and the amount of evasion. The substitution effect in the A-S model occurs because the penalty rate is held fixed when the regular tax rate increases, so that the difference between the penalty rate and the regular tax rate goes down, and this increases the incentive to underreport income. In the Yitzhaki reformulation this effect vanishes.

The absence of ambiguity in theoretical models is often considered to be a good thing, but there is a paradox involved in the Yitzhaki analysis. The ambiguity of the original A-S analysis is removed, but what is left is a result that goes directly against most people’s intuition about the connection between the marginal tax rate and the amount of evasion. It seems to be a common belief that high marginal tax rates encourage tax evasion because there is a large gain to be made from withholding income from the tax collector. This belief is inconsistent with the Yitzhaki formulation, for there the penalty increases pari passu with the tax rate. In the A-S model, by contrast, there is a positive substitution effect on evasion because the net penalty - the difference between the penalty rate and the regular tax rate - goes down when the tax rate increases. It is worth noting that this substitution effect would be present under the more general but weaker assumption that the penalty rate increases less than proportionately with the tax rate. Perhaps the theoretical ambiguity in this case is more representative of popular beliefs and possibly even of actual tax systems.

5. Endogenous detection probability.

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7 The study by Clotfelter (1983) of tax return data for the United States found a strong positive association between marginal tax rates and the amount of evasion. For surveys of empirical work in this area see Schneider and Enste (2000) and Slemrod and Yitzhaki (2002).
The A-S article also considered the case where the probability of detection varies with the amount reported\(^8\), so that \( p = p(W-E) \). Under the assumption that \( p'(W-E) < 0 \), it was shown that the predictions of the model concerning the effects of an increase in the penalty rate and a positive shift in the probability of detection function continued to hold; a higher penalty and a higher likelihood of being detected both act as a deterrent to tax evasion.

The probability function must be taken to reflect the taxpayer’s beliefs about the policy followed by the tax collection agency. How is this belief justified? A-S discussed both alternatives as to the slope of the \( p(W-E) \) function. In the absence of any information about \( W \), it might be reasonable to assume \( p'(W-E) > 0 \), assuming that the agency believes that the rich tend to evade more. On the other hand, to assume that the agency is completely ignorant about \( W \) may be unrealistic. E.g., it could reasonably be expected to know the taxpayer’s profession and the normal level of income associated with it. Then, for each type of profession, the natural hypothesis would be \( p'(W-E) < 0 \); as reported income approaches or exceeds the normal level, the probability of detection falls.

This is a little ad hoc. A more satisfactory approach is to derive a rule from a model of policy optimization on the part of the agency. This has been done in a number of more recent papers beginning with Reinganum and Wilde (1985), which have been very carefully surveyed by Andreoni, Erard and Feinstein (1998). In this literature it is usually assumed that the collection agency’s objective is to maximize expected tax revenue, although this is a rather special assumption seen from a welfare economics point of view; see the discussion in Section 11 below.

In the case where the tax authority can commit to an audit rule before taxpayers report their incomes, a central result established by Reinganum and Wilde (1985) is that the optimal policy involves a cut-off point whereby all returns reporting an income below some critical level are audited with probability 1, whereas those who report an income higher than this are not audited at all. This equilibrium rule can be interpreted as an extreme version of the A-S assumption, although it is based on the simplification that taxpayers are risk neutral. When a commitment to an audit rule cannot be made, as in Reinganum and Wilde (1986), the analysis gets much more complicated and takes the form of a sequential move game with a number of

\(^8\) Somewhat curiously, a large number of later contributions state rather emphatically that it neglected the problem.
possible equilibria. It is difficult, on the basis of these analyses, to arrive at reasonably simple and testable conclusions about the shape of the probability of detection function. Perhaps the main value of this type of analysis will be as a basis for internal analysis of optimal policy choices in tax collection agencies.

6. A cynical view of taxpayer behaviour?

The model in Section 4 assumes that the taxpayer does in fact engage in tax evasion; the model is one with an interior solution in which $E>0$. But is it clear that the taxpayer will always choose to evade taxes? In other words, is it always optimal to move from a state of no evasion ($E=0$) to one with a positive evasion level? To see what the model says about this, we take the derivative of expected utility at $E=0$, where $Y=Z=(1-t)W$, to obtain

$$\frac{\partial V}{\partial E}\bigg|_{E=0} = (1-p)U'((1-t)W)t-pU'((1-t)W)(-t).$$

For some tax evasion to be optimal, it must be the case that this derivative is positive. It is easily seen that this is the case if and only if

$$p < t.$$  \hspace{1cm} (7)

For tax evasion to be optimal from the taxpayer’s point of view, it is a necessary and sufficient condition that the expected penalty rate is less than the regular tax rate.

This condition presents us with something of an empirical puzzle. Obviously, we do not know how many people evade taxes, but it is fairly certain that there are a large number of people who do not, even though they have the opportunity to do so. The behaviour of these people can only be explained by the model if one assumes that for them the inequality (7) is reversed. Is this reasonable? If, to take an illustrative example, the penalty rate is twice the regular tax rate, (7) implies that the probability of detection which is sufficiently high to deter tax evasion is greater than 0.5. This number is far in excess of most empirical estimates and raises the
question of whether the model depicts people as either too rational or too cynical compared to what we believe that we know about their actual behaviour.\textsuperscript{9}

There are several ways to resolve this puzzle. Thus, it is important to realize that the probability that enters condition (7) is the taxpayer’s subjective probability, which is not necessarily equal to the statistical frequency with which peoples’ tax returns are checked. Indeed, empirical studies indicate that people tend to overestimate the probability of detection, and this could go some way towards explaining non-evading behaviour. (For an evaluation of these studies see Andreoni, Erard and Feinstein (1998, pp. 844-847).)

But this explanation is not entirely convincing. Common sense and everyday observations tell us that people refrain from tax evasion - as well as from speeding, shoplifting and polluting the environment - not only from their estimates of the expected penalty, but for reasons that have to do with social and moral considerations.\textsuperscript{10} Allingham and Sandmo (1972) considered the social stigma that may be attached to being caught in evading taxes and showed that this leads to a more restrictive condition for tax evasion to be optimal. Another alternative is to assume that people have a bad conscience about evading taxes. A simple representation of this is to let the “disutility” of tax evasion be represented in the expected utility function\textsuperscript{11}, so that we can write

\[ V = (1-p)U(Y) + pU(Z) - B(E). \] (8)

Here it is assumed that \( B'(E) > 0 \) and that \( B''(E) > 0 \). An important insight is that this extension of the analysis leaves the qualitative nature of the comparative statics effects for the interior solution case completely unaffected. The reason is simple. First, the budget constraints remain the same with this extension. Second, the concavity of expected utility, \( V \), in the decision variable \( E \) is also unaffected. These two facts together mean that the qualitative predictions of the model must be the same as before. What changes, however, is the condition for positive tax evasion to be optimal, which now becomes

\textsuperscript{9} Frey and Feld (2002) mentions this as a major empirical problem with the original A-S analysis. These authors estimate the probability of detection to be substantially less than one per cent on the basis of data for Switzerland, 1970-1995.

\textsuperscript{10} For further discussion see McCaffery and Slemrod (2004), who discuss this issue in the wider context of behavioural public finance.

\textsuperscript{11} A similar model can be found in Gordon (1989), who assumes that the function \( B \) is linear.
This inequality implies that a positive expected gain is not sufficient for the taxpayer to engage in tax evasion. The negative value attached to evasion as such acts as an additional penalty - a “conscience tax” - to deter evasion.

An interesting implication of this formulation is that it leads to a less optimistic view of the effectiveness of using penalty taxation as deterrence to tax evasion. In the new version of the model it will still be true that an increase in the penalty rate leads to less evasion. But because evasion decreases, the “conscience tax” \( B'(E) \) also goes down, and this diminishes the effect of the penalty tax. In other words, the stronger extrinsic incentive to truthful reporting reduces the intrinsic incentive to behave honestly. Some writers have maintained that this type of behavioural reaction cannot be explained within the framework of neoclassical economic theory, but clearly this is not true. The present extension of the model does seem to capture the common notion that explicit economic incentives make taxpayers see it as less imperative to act according to moral standards of behaviour. On the other hand, one may question whether this is a very meaningful empirical assumption. What we can hope to observe is the net effect of the tax increase – and even that is obviously very difficult. A further empirical decomposition with the aim of discovering the change of balance between intrinsic and extrinsic incentives is fraught with severe difficulties.

Expected utility maximization has come under increasing criticism in recent years as not being in accordance with observed behaviour in a number of areas, and another modification of the A-S model consists in replacing the expected utility framework with an alternative axiomatic model of choice under uncertainty. An example of this is the work of Eide (2002), who replaces expected utility with so-called rank dependent expected utility (RDEU). It turns out that this does not change the comparative statics results for the person who evades tax. What it does change is the condition for some tax evasion to be optimal, which becomes more restrictive in the RDEU model. In this respect it is similar to the variations on the expected utility model that consist in introducing social stigma or a bad conscience. This suggests, as indicated above, that the comparative statics results of the A-S model may be rather insensitive to changes in assumptions about the taxpayer’s objective function. A bigger challenge to the model is to introduce a larger choice set for the taxpayer, e.g. by studying the interaction between tax evasion and labour supply.
7. Tax evasion and black market work.

The A-S model looks at the tax evasion decision in isolation from other types of economic choices. This may be an advantage because it leads to clear and reasonably unambiguous hypotheses, but at the same time it seems clearly unrealistic. It is unreasonable to believe that the taxpayer has not thought about the possibility of evading taxes before he sits down to fill in his income tax return. More probably, he has thought about this before making decisions about the allocation of his work and leisure hours or about the composition of his investment portfolio. It is especially the connection between tax evasion and labour supply that has caught the interest of economists working in this area. This is in many ways natural, since public attention to the hidden economy has focused on this problem, presumably because, in spite of its name, it is far from hidden from the majority of people, many of whom have personal experience with the black labour market, either as sellers or buyers of labour services.

Once we link the amount of tax evasion with the earnings from black market work, it is clear that we can no longer take the view of the A-S model that decisions about tax evasion can be postponed to the moment of filling out the income tax return. Instead, such decisions are closely related to labour market choices, so that the tax evasion decision should be studied in the context of a model of labour supply. In the model of Section 4, utility depended only on income, which, in a one-period world, is equivalent to consumption. The first extension of the model must be to allow for the labour-leisure choice, so that we rewrite the basic utility function as $U(C, L)$, where $C$ is consumption and $L$ is leisure. The utility function is assumed to have the usual convex indifference curves and also to be concave in $C$, so that the taxpayer is risk averse towards variations in his consumption level. Let us now interpret $Y$ and $Z$ of equations (1) and (2) as the levels of consumption in the two states of non-detection and detection. Expected utility can then be written as


But the constraints also differ from the previous formulation and can in fact be formulated in several different ways. The one that is closest in spirit to the identification of tax evasion with
black market work, starts with a time constraint that lets total time available consist of three parts, regular market hours, black market hours and leisure; this can be written as

\[ H + h + L = T. \]  \hspace{1cm} (11)

Gross income is then

\[ W = w_0H + w_1h, \]  \hspace{1cm} (12)

where \( w_0 \) is the wage rate in the regular labour market and \( w_1 \) is the wage paid for black market work. The final step of the extension is to identify black market earnings with the amount of income evaded, so that

\[ E = w_1h. \]  \hspace{1cm} (13)

Consumption in the two states of non-detection and detection can then be written as

\[ Y = (1-t)w_0H + w_1h, \]  \hspace{1cm} (14)

\[ Z = (1-t)w_0H + (1-?)w_1h. \]  \hspace{1cm} (15)

Note that we take the two wage rates as exogenously given; we return to the question of tax incidence below.

Maximizing expected utility (10) subject to the budget constraints (14) and (15) and the time constraint (11) leads to first order conditions that can be subjected to comparative statics analysis. I will not go into the details of this, which easily gets quite complicated. Instead I will simply summarize a few insights that one can derive from the model and contrast them with the results from the original A-S model.

The present model is a mixture of the standard textbook model of labour supply and the A-S tax evasion model. In general, we must expect the composite model to yield less clear-cut results than the partial models, while at the same time alerting us to connections that are less obvious in a more partial view. Some of the results from the partial models turn out to carry
over to a more general setting. Thus, the effects of changes in the penalty rate and the probability of detection turn out, on reasonable assumptions, to have the same signs as they have in the A-S model; an increased penalty rate as well as an increased risk of detection both tend to deter tax evasion. Moreover, the compensated effect on regular labour supply of a change in the marginal tax rate is negative, just as in the standard model. Does a high marginal tax rate encourage black market labour? Here it becomes even clearer than in the A-S model that the theoretical answer must necessarily be ambiguous. Even if we neglect the income effects, a reduction of hours in the regular market must be accompanied by increased hours *either* in black market labour or in leisure, but the model cannot predict how the increased hours will be divided between the two types of activity. The marginal tax rate effects on evasion and black market activity remain indeterminate in the sense that no clear empirical hypothesis emerges from the theoretical model.

A different type of model was formulated by Pestieau and Possen (1991), who studied the connection between tax evasion and occupational choice. Individuals can choose between being wage-earners, who have no opportunities for tax evasion, and entrepreneurs, who do. The stricter the enforcement of the tax law becomes, the smaller becomes the fraction of the population who chooses to become entrepreneurs. A rather different perspective on this issue is in Kolm and Larsen (2004), where only manual workers are assumed to have access to the black labour market. In their model stricter measures against the black labour market lead more manual workers to take education. These may appear to be rather different stories about tax evasion and occupational choice, but how different they are, obviously depends on how one identifies real-life occupations with theoretical concepts like “entrepreneurs” and “manual workers”. The general message is clearly that a stricter enforcement policy may affect tax compliance via the incentives to choose occupations where the opportunities to evade taxes or engage in black market activities are less.

8. **Incidence effects of the black economy.**

In an otherwise competitive setting, it is natural to think about the incidence effects of the black economy in the following manner: When firms and consumers hire black market labour, 

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12 This is based on an interpretation of the results in Sandmo (1981, especially pp. 269-275), although the interpretations are not spelled out in detail there.

13 A similar conclusion was reached by Pencavel (1979) on the basis of a model with the special assumption of additive preferences but with a more general income tax schedule. See also Baldry (1979).
it must be because it is cheaper; gross wages must be lower than in the regular economy. Moreover, when individuals supply labour to the black economy, it must be because it pays them to do so; net wages must be higher than in the regular economy. Consequently, we should expect the private gains from tax evasion to be divided between employers and workers, and the more precise nature of this division must depend on the use of the government’s policy instruments. This incidence problem, although important and interesting, has so far received little attention in the literature. I shall just provide a brief sketch of how one could go about thinking about the subject.

First of all, it is not clear that standard competitive assumptions are quite appropriate for a study of this area. Workers may be constrained in their labour supply, either because of regulations on hours in the regular market or because they are unemployed. In either case, the representative consumer is not the one we have considered so far, who on the margin equalizes the advantages from both types of employment. A further complication is that agents in the market may not always know whether or not they find themselves in the black economy; it may not always be clear to me as a buyer of some service whether the cheap offer is the sign of a competitive supplier or whether he has some hidden strategy for tax evasion. Finally, the incidence must also depend on the legal responsibility for truthful reporting; does this rest only on the seller of black market labour, or does the buyer also carry some share of it?

These questions mainly represent a list of unsolved problems. In the following I shall revert to the competitive set of assumptions and ask a more limited question: What determines the relative wage rates in the two sectors that make the taxpayer/worker indifferent between working in the two sectors? In the one consumer economy this will be the wage rates that guarantee an interior solution. In a more general analysis with many taxpayers with different productivities and market opportunities we would expect to see an equilibrium where some work only in one of the two sectors, while others share their working time between the two. The last of these groups should on the margin be indifferent between the two types of work, just as our representative taxpayer, and it is this indifference at the margin which gives us the clue to understanding equilibrium wage rates.\footnote{In the interesting paper by Kesselman (1989) individuals are assumed to make an all-or-nothing choice with respect to the black economy, and the risk aspects of the choice are not modelled explicitly.}
In the model of the previous section it was assumed that from a consumption point of view the nature of the work is the same; the consumer cares only about his number of leisure hours, and if the pay were the same, he would be indifferent between working in the two sectors. But the concept of “the same pay” is not a simple one in the present context. It should obviously refer to the after-tax situation of the taxpayer, and this has to be represented by taking account of all the policy parameters and the uncertainty aspects of the equilibrium.

We approach this problem in a stepwise fashion, beginning with the simplest possible case. If the probability of detection had been zero, we would have expected to observe an equilibrium where the relative wage rates had been characterized by the condition

\[(1-t)w_0 = w_1.\] \hspace{1cm} (16)

In this case the virtual tax exemption for earnings in the black labour market would be fully capitalized in the wage rates with \(w_0 > w_1.\)

With a positive probability of detection and risk neutrality, we would have the equilibrium condition

\[(1-t)w_0 = (1-p^*)w_1.\] \hspace{1cm} (17)

We would still have \(w_0 > w_1\) (recall that \(p^* < t\) for an interior solution) but the equilibrium wage rates will be closer to each other, since there is, after all, a positive expected tax rate in the black economy.

When we finally move to the case of risk aversion that we considered in the previous sections, it turns out that for an interior solution we must require that

\[(1-t)w_0 = (1-\beta p^*)w_1.\] \hspace{1cm} (18)

Here \(\beta\) is a risk adjustment factor given by

\[\beta = U_Z(1-p)U_Y + pU_Z.\] \hspace{1cm} (19)
Risk aversion - concavity of $U$ - implies that $U_Z > U_Y$, so that $\beta > 1$. Looking back at equation (18) we see that the effect of $\beta$ can be interpreted as “blowing up” the probability of detection and make the expected penalty appear larger. This makes the equilibrium wage rates move even closer together. The gap between the regular and the black market wage rates widens with the regular tax rate and narrows with the penalty rate, the probability of detection and the degree of risk aversion (as measured by the curvature of the marginal utility of consumption). In brief, the wage rate in the black economy must compensate the worker for his risk-taking.

This is a very incomplete analysis of tax incidence. In particular, it does not explain how it is that firms and consumers who demand labour services will come to employ both regular and black labour in the face of substantial differences in gross wages. There may in fact be some disadvantages to employing labour from the black market. Firms that do it on a regular commercial basis may violate the law and run the risk of legal sanctions. Private consumers who employ black labour to paint their house or repair their car may realize that they have less security of contract than in the regular market, and this may make them unwilling to pay the full regular market wage to black market labour. In a more general incidence analysis these considerations should be integrated with the analysis above. A more general analysis should also go beyond the labour market and study the effects of the black economy on relative prices of goods and services: Does competition between the two sectors drive down the prices of services that are typically produced in the black economy? Kesselman (1989) studies such problem in a two-sector general equilibrium model in the Harberger tradition and finds that relative price effects tend to modify the effects on evasion of changes in the marginal tax rate on income.

Of more importance than the effects of evasion on the functional distribution of income may be the effects on the personal distribution. The A-S model suggests that evasion increases with gross income, while the effect on the fraction of income evaded depends on relative risk aversion (Allingham and Sandmo, 1972 p. 329); e.g., if relative risk aversion is constant, the fraction of income evaded will be constant. However, this result assumes that the perceived probability of detection does not vary with income. There is empirical evidence to indicate that evasion is higher from income from independent business than for wage income, and it may also be the case that high-income people spend more resources on efforts to conceal their
true income. If this is true, the presence of tax evasion could change a proportional statutory income tax into a regressive one or reduce the effective progressivity of a progressive tax.\(^\text{15}\)


The models surveyed so far all concern evasion by individual taxpayers, while the role of firms has been very much in the background. I have briefly noted the possible role of firms in the black labour market, but firms could also have a more independent role in tax evasion activities as evaders of indirect taxes for which they act as tax collectors for the government. This role was first studied in a theoretical framework by Marrelli (1984), who extended the A-S model to the case of a risk-averse firm and established comparative statics results similar in nature to those of the A-S model.\(^\text{16}\)

Marrelli explicitly studied the case of an ad valorem tax which was not seen in the context of the value-added system. The value-added system has a self-policing property in that buyers of intermediate goods have opposing interests to the sellers, and this reduces the scope for indirect tax evasion. The main problem with indirect tax evasion may therefore be at the final stage of production, i.e. at the sale of final goods and services to consumers. It is also of potential importance for areas like environmental taxes, where taxes on emissions may be evaded by the polluter; for details and references to the earlier literature on this see Sandmo (2002).

There is one result in the theory of firm evasion of indirect taxes which is particularly interesting and concerns the separability of production and evasion decisions. This can be illustrated in a simple model of a competitive firm which produces a single output in the amount \(x\), sells it at a tax-inclusive price \(Q\) and pays a specific tax \(t\) per unit of output. It reports sales of \(x-e\), where \(e\) is the amount of underreporting. If discovered, the firm has to pay a fine \(?'(e)\) on the amount evaded such that \(?'(e)>t\) (the marginal fine must be greater than the tax), and \(?''(e)>0\), so that the marginal fine is increasing. Assuming that the firm is risk neutral, expected profit is

\(^\text{15}\) E.g., in the model of Persson and Wissén (1984), which is a direct extension of the A-S model, there is a linear progressive tax system; in spite of this taxation may become regressive when evasion is taken into account.

\(^\text{16}\) Marrelli and Martina (1988) is a further extension of the results to an oligopolistic market with strategic interaction between firms.
\[ ep = (1-p) \{ Qx - c(x) - t(x-e) \} + p \{ Qx - c(x) - t(x-e) - ?(e) \}, \] (20)

where \( t \) is the indirect tax rate, \( c(x) \) is the cost function and \( p \) as before is the probability of detection. Taking the derivatives of expected profit with respect to \( x \) and \( e \), the first order conditions for this problem can be written as

\[ Q = c'(x) + t. \] (21)

\[ p?'(e) = t. \] (22)

The interesting feature of the firm’s optimum is that the output decision is independent of the probability of detection and the penalty function; equation (21) determines output, while (22) determines the amount of underreporting. An implication of this is that if the tax rate \( t \) has been set with the aim of achieving some specific policy objective, e.g. to achieve some reduction of the consumption of a good with negative external effects, the optimal tax rate is unaffected by the opportunities for evasion. The tax rate should be used to achieve the right consumer price, while evasion should be controlled by the probability of detection and the penalty function. It is interesting to note that a similar result holds if the firm is risk averse and maximizes the expected utility of profit instead of just expected profit. In that case condition (21) still holds; it remains optimal for the firm to set marginal cost, inclusive of tax, equal to the consumer price. Risk aversion only modifies condition (22), making the risk averse firm evade less than its risk neutral counterpart for any given level of output.

Firms may not only evade the payment of indirect taxes; they may also evade corporate income taxes, and this is possibly of even greater practical significance. This problem has been examined in recent papers by Chen and Chu (2002) and Crocker and Slemrod (2003). These authors point out that the theoretical framework of the A-S model is likely to be inadequate in this context, since the separation of ownership and control is essential for understanding corporate tax evasion. They therefore explicitly model the tax evasion decision in the context of the contractual relationship between the shareholders and the manager of the corporation. An interesting result that emerges from the analysis is that the effect of policies to control evasion may depend crucially on who is penalized, the corporation or the manager. This type of result has no counterpart in the previous literature and indicates that this may be a promising new area of research.
10. From the individual to the aggregate: Tax evasion in a social context.

Most of the theoretical literature on tax evasion considers the decisions of a single individual, and empirical predictions about the extent of tax evasion take their points of departure from hypotheses derived from the comparative statics of the model of the individual taxpayer. A more careful analysis would take the aggregation problem more seriously and base predictions on a model of many taxpayers, differing both with respect to their income and evasion opportunities. But even such a model might not take adequate account of tax evasion as a social phenomenon. There is empirical evidence to indicate that the amount of tax evasion and black market behaviour differ substantially between countries\(^{17}\), and it is far from obvious that the differences can be satisfactorily explained by tax and penalty parameters. Cowell (1990, ch. 6) considers a number of alternative explanations of tax evasion as a social phenomenon, requiring a theory of social interaction. I will illustrate by considering two channels through which tax evasion behaviour might spread in a population of taxpayers.

In the A-S model with an endogenous probability of detection it is assumed that the probability of detection increases with the amount that one evades. But how does the taxpayer form his views about this probability? One source of information about the probability is the taxpayer’s own observations of other taxpayers; he may have observed neighbours or colleagues who work in the black economy and apparently get away with it. This means that we could write the taxpayer’s subjective probability of detection as a function both of his own evasion and of his perceived amounts of evasion by others. Suppose now that his perception of the amount evaded by others increases. His subjective probability of detection falls, and he therefore decides to evade more. Others now perceive that he evades more and therefore evade more themselves. It is not difficult to see that this kind of social process could involve multiple market equilibria, either with high or low evasion, with only small differences in the tax and penalty incentives.

Another mechanism that could lead to similar results comes from the social conscience effect described in Section 6. There is a disutility involved in evading taxes, but it might be smaller if it is perceived that many others evade taxes also. If individual \(i\)’s perception of the evasion

\(^{17}\) For a recent survey and further references see Schneider and Klinglmair (2004).
of others were to increase, e.g. as a result of increased attention to these matters in the media, he might decide to evade more, which could again trigger more evasion by others. The same is true of the social stigma effect of the original A-S model; the stigma attached to being caught for tax evasion is less if this is a common occurrence.

This sketch of an analysis has elements in common with the analysis of corruption by Andvig and Moene (1990). The main point of their model is that it is individually more costly to be honest in a country where corruption is common. Similarly, it may be less risky to evade taxes in a country where evasion is widespread. At a more general level, the analysis is related to that of Schelling (1978), who studies externality models with multiple equilibria in which small changes in parameter values may lead to large changes in the equilibrium values of the endogenous variables.

11. Issues of optimal taxation.

The two main elements in a model of optimal taxation are the social welfare function and the government budget constraint. The optimal taxation problem is posed as that of choosing a set of tax instruments that maximize social welfare subject to a government budget constraint. The budget constraint often takes the simple form of requiring a given tax revenue; more generally, tax revenue should be equal to government expenditure on goods and services, which could also be subject to optimization within the same model.\(^\text{18}\)

To extend the usual analysis of optimal taxation to take account of tax evasion raises some problems for both of these elements. As regards the social welfare function, the problem arises in regard to the principle of consumer sovereignty or the Pareto principle. We usually take it for granted that the social welfare function should be increasing in individual utilities. Now suppose that there occurs an exogenous “improvement” - from the point of view of the tax evader - in the opportunities to evade taxes, so that the probability of detection falls. The immediate effect of this is to raise the expected utility of all tax evaders. Should this count as a welfare improvement? It is obviously realistic to assume that increases in the expected utility of the tax evaders will be accompanied by decreases in utility on the part of the non-

\(^{18}\) An early contribution in this area is Kolm (1973), who wrote of the A-S analysis, "...this is hardly public economics; in fact, it is very private." Extending the analysis to take account of the utility of public goods, Kolm studied the government’s optimal choice of deterrence instruments subject to a government budget constraint.
evaders; in this particular example the non-evaders might e.g. have to pay more taxes. But the basic issue remains the same: When aggregating welfare gains and losses across individuals, should the gains from evasion count on a par with the gains from other economic activities? Some thought will have to be given to this problem before one formulates a model of optimal taxation with tax evasion.

The problem with regard to the government budget constraint is of a more technical nature and lies in the fact that the standard formulation consists in assuming that tax revenue should be equal to public expenditure. If taxpayers try to evade taxes and their evasion is only discovered with some probability, it implies that tax revenue will be uncertain also. An apparent solution to this problem is to write the government’s budget constraint in terms of expected values, but this is inconsistent with a general equilibrium formulation. If \textit{ex post} it turns out that revenue is less than expenditure, then the deficit must be financed in some way, e.g. by tax increases or public borrowing. But then those methods of finance should have been part of the optimization problem from the start.

The essence of models of optimal taxation is to clarify the respective roles of equity and efficiency considerations in the design of tax systems. A framework in which the equity-efficiency trade-off becomes particularly transparent is that of choosing an optimal linear income tax, as in Sandmo (1981). The model has two groups of taxpayers, the evaders (or, more accurately, the \textit{potential} evaders) and the non-evaders. The non-evaders’ behaviour is modelled as in the standard textbook analysis of labour supply, while the evaders are described by the model of Section 6 above\textsuperscript{19}. The social welfare function is the utilitarian weighted sum of utilities, where the weights can be varied to give more or less importance to the utility of the evaders. The problem related to the government budget constraint is solved by assuming that the evaders’ subjective probability of detection is equal to the actual frequency of audit - an empirically dubious but analytically useful simplification. There is assumed to be an exogenous revenue requirement, and in addition the government needs resources to cover the administrative costs of the tax system, which in this context is simply equal to the costs of discovering tax evasion. Formally, the model can be written as follows.

\textsuperscript{19} Cremer and Gahvari (1994) have an alternative formulation of the model in which they assume that taxpayers can spend resources on lowering the probability of detection, and this is reflected in their formula for the optimal tax rate.
Choose the parameters of a linear income tax, the penalty rate and the probability of detection that maximize the social welfare function

\[ W = N^n \gamma^n V^n + N^e \gamma^e V^e, \]

where \( N^n \) and \( N^e \) are the numbers of non-evaders and evaders, \( V^n \) and \( V^e \) are their expected utilities, and \( \gamma^n \) and \( \gamma^e \) are the social welfare weights. The maximization is subject to the government budget constraint, which is

\[ R = R^0 + f(p)N^e. \]

Here \( R \) is revenue as a function of the policy instruments, \( R^0 \) is the fixed revenue requirement, and the last term is administrative cost as a function of the frequency of audits (which again is equal to the probability of detection).

What are the types of question that this model can answer? Perhaps the most immediate question would be whether the optimal marginal tax rate is lower because of tax evasion. Implicitly at least, this is often maintained in policy discussions when it is claimed that a higher tax rate encourages evasion. Here we already know from the model of the individual taxpayer that it does not offer a clear prediction, but even if one believes that a high marginal tax rate encourages evasion, the optimum tax analysis does not yield a clear conclusion in favour of a lower tax rate.\(^{20}\) The clue to understanding this ambiguity is the insight that the black labour market is less distorted than the regular one, and a tax increase that pushes labour from the more distorted to the less distorted sector may represent an efficiency gain to the economy. From one point of view, this makes economic sense. However, it also raises some question about the fundamental assumptions of a model that turns “antisocial” behaviour into a social gain. These are to be found in the social welfare function, but it is not easy to see which alternative assumptions would lead to more palatable conclusions. Slemrod and Yitzhaki (2002), commenting on a similar discussion by Cowell (1990), conclude that “in our opinion no convincing alternative that provides reasonable policy prescriptions has yet been presented” (p. 1447).

\(^{20}\) Cremer and Gahvari (1994) show that the optimum marginal tax rate is lower with tax evasion, provided that the social welfare function displays inequality aversion, but the result depends on their assumption that the individual utility function is quasi-linear in consumption, so that taxpayers are risk neutral. See also the analysis of this issue in Schroyen (1997).
How high should the penalties be? One possible answer to this is that the penalty rate and the frequency of audits should be chosen so as to maximize tax revenue. In the present model this prescription turns out to be correct only for the special case of $\gamma = 0$. This is the case where no weight is given to the evaders’ utility, and the optimal tax problem is to choose that policy which maximizes the utility of the non-evaders. This case is clearly not without interest. If you believe that it is the rich who evade taxes, you might take the Rawlsian view that anti-evasion policy should aim to maximize the utility of the poor non-evaders. The non-evaders’ interest in deterring evasion lies in the net revenue to be got from the penalties; the higher this is, the lower will the regular income tax be. But in a more general analysis, where the evaders’ utility is taken into account, the conclusions must be modified to lower both the penalty rate and the frequency of audit from their revenue-maximizing levels.

There is also an issue of horizontal equity involved here. Suppose that you believe that people with income from independent business are more likely to evade income, so you identify them with the evaders of the model, in contrast to the non-evading wage earners. One reason for not trying to extract a maximum of tax revenue from the former group is that by so doing you might come to inflict heavy punishment on a small subgroup of people for violations of the tax law committed by a much larger group. This might have some positive incentive effects, but if one has inequality aversion it is also a policy with rather unattractive equity characteristics.

This discussion of the optimal taxation problem has solely been concerned with income tax evasion. The optimal indirect tax problem has been studied by Cremer and Gahvari (1993), who show how the Ramsey conditions have to be modified when the extent of tax evasion differs across commodities. Under some conditions, they show that the optimal statutory rate will be higher for commodities subject to evasion; however, the optimal effective (or expected) rate is lower for commodities subject to tax evasion.

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21 Alternatively, one might conceivably take the view that only the welfare of the honest should count in the social welfare function.
22 Penalties could also be set above their revenue-maximizing level, with unfortunate consequences for social efficiency, as pointed out by Adam Smith in Book V, Part II, of the Wealth of Nations: “...by the forfeitures and other penalties which those unfortunate individuals incur who attempt unsuccessfully to evade the tax, it may frequently ruin them, and thereby put an end to the benefit which the community might have received from the employment of their capitals” (Smith 1776; 1976, p. 826).
23 The equity aspects of tax enforcement have been analyzed in more depth by Boadway and Sato (2000)

One should obviously be careful about drawing policy conclusions from the theoretical literature in an area where our empirical knowledge is by necessity so uncertain. Let me nevertheless hazard some tentative conclusions.

The first is based on the theoretical insight that the probability of detection (the frequency of audits) and the penalty for evasion are policy substitutes. If one wishes to achieve a given degree of deterrence, this may be achieved by high probabilities and low penalties or by low probabilities and high penalties. The concern for low costs of tax administration leads one to favour the second alternative; this was Becker’s (1968) argument in his general analysis of the economics of crime. However, such a policy might lead to unacceptable high penalties for a few for violations committed by many, a horizontal equity argument neglected by Becker\textsuperscript{24}. A counterargument might be that one could then just set penalties so high that nobody would evade taxes. But for penalties to be socially acceptable, they probably must be set so that in the eyes of the general public, they “fit the crime”. In this regard, the optimum tax analysis of Section 11 may be a bit simplistic in assuming that the optimal penalty can be set independently of the regular tax rate.

Is the existence of tax evasion an argument for a lower marginal tax rate? We have seen that the optimal tax analysis does not offer any clear conclusion on this point, and my own inclination is to say that, at least as long as tax evasion is not an overwhelming social problem, the choice of the marginal tax rate should be governed by the more standard efficiency and equity concerns. The penalty and audit rate are instruments better targeted on the decision to evade taxes.

The tax evasion decision may depend on the individual taxpayer’s perceptions of the behaviour of others. The more widespread evasion is, the more socially acceptable it may become, and the lower is the subjective probability of being detected. This is a good reason for trying to control evasion; relaxing policy measures in this area may unleash mechanisms that lead to a much higher level of tax evasion.

\textsuperscript{24} An additional complication is that some violations of the law may be committed by ignorance and mistakes, a point made by Stern (1978) in a general evaluation of the Becker model.
Appendix

This appendix derives the expressions for the income effect on tax evasion and the tax rate
derivative in equation (6).

Starting from the first-order condition (4), we note that the second-order derivative has to be
negative for a maximum. This implies that

\[(1-p)U''(Y)t^2 + pU''(Z)(\bar{r} - t)^2 = D < 0.\]  \hspace{1cm} (A1)

Now differentiate the first-order condition with respect to \( W \):

\[(1-p)U'(Y)[(1-t)+t(\partial E/\partial W)]t-pU'(Z)\{[1-t]+(\bar{r}-t)(\partial E/\partial W)\}(\bar{r}-t)=0.\]  \hspace{1cm} (A2)

Collecting terms and inserting \( D \) as defined by (A1), we can rewrite this expression as

\[D(\partial E/\partial W) = -(1-p)U'(Y)(1-t)t[R_A(Y) - R_A(Z)].\]  \hspace{1cm} (A3)

Define the coefficient of absolute risk aversion, \( R_A \), as the negative of the ratio of the second
and first derivative of the utility function; for the properties of this coefficient as a measure of
risk aversion see Arrow (1974, Essay 3). Thus, we have that

\[R_A(Y) = -U''(Y)/U'(Y) \quad \text{and} \quad R_A(Z) = -U''(Z)/U'(Z).\]  \hspace{1cm} (A4)

Using this definition and the equality of the two terms in (4), we can rewrite (A3) as

\[\partial E/\partial W = (1/D)(1-p)U'(Y)(1-t)t[R_A(Y) - R_A(Z)].\]  \hspace{1cm} (A5)

Decreasing absolute risk aversion implies that the expression in brackets is negative, so that
\( \partial E/\partial W > 0 \), proving the assertion in the text.

The expression (6) for the tax rate effect is derived by differentiating equation (4) with respect
to \( t \). We then obtain
\[(1-p)U'(Y)+pU'(Z)+(1-p)U''(Y)[-W+E+(?E/?t)]t-pU''(Z)[-W+E-(?E/?t)(?E/?t)](-t)=0.\]  
(A6)

Inserting the expression for \(D\) and collecting terms, we have that

(A7)

We now substitute in the first term on the left for the expression for \(?E/?W\) in (A3). Defining

\[S=-(1/D)[(1-p)U'(Y)+pU'(Z)],\]  
(A8)

we then get

\[?E/?t=-(W-E)/(1-t)(?E/?W)+S,\]  
(A9)

which is equation (6) in the main text.

The expressions for the effects of increases in \(?\) and \(p\) can be derived by using the same procedure as above; this is left as an exercise.
References


