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A Solution to Tax Evasion

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Abstract

Indirect (unit) tax collection is difficult because it is hard for the government to monitor the economic dealings. In most countries, the firm incurs cost to cheat the government while so does the government to audit the firm. To solicit the information on a transaction known only to the firm and the consumer, several governments have set up a cashback system.[1] The paper shows if the government gives a subsidy to the consumer in a competitive market, the consumer will voluntarily and fully declare the tax so that the firm cannot cheat, and that the cheating and auditing costs can be saved; Pareto improving and efficient taxation without evasion becomes practicable.

JEL classification: H26, D81, D82

Keywords: tax evasion, unit tax, subsidy, tax declaration

[1]For example, Armenia, Bolivia, Mainland China, Northern Cyprus, Indonesia, Korea, Philippines and Taiwan have used the cashback or lottery receipt system. See the first section of this paper for details.

1 Introduction

To collect indirect taxes, such as the unit tax, the government needs to obtain private and corporate financial records of transactions. However, unless the government is willing to pay the significant cost of monitoring the economic dealings and collection processes, such information will not materialize. Owing to the asymmetry of information between the government and taxpayers, the taxpayer is tempted to underreport the due tax amounts.

The Literature

Allingham and Sandmo (1972) and Yitzhaki (1974) are pioneers in analyzing how a risk-averse taxpayer chooses an optimal unreported income to maximize the expected utility under the governmental auditing. Since then there are enormous theoretical and empirical studies on tax evasion. Andreoni et al. (1998) make a comprehensive survey of this literature. It has been theoretically found that tax enforcement, auditing, tax rate, income level and social norms, etc. should have impacts on tax evasion. Cowell (1990) lists the empirical studies on the extent of tax evasion and the black economy in nineteen major countries. Crane and Nourzad (1994), O'Higgins (1989), and Alm et al. (1991) emprically examine the determinants of income tax evasion in the U.S., U.K., and Jamaica. Slemrod (2007) offers recent reviews of the literature. Marion and Muehlegger (2008) examine tax evasion in the diesel fuel market in U.S. Gorodnichenko et al. (2009) examine the effect of Russia's 2001 flat rate income tax reform on tax evasion.

Ishi(1981) points out the issue of "kuroyon" in Japan, which refers to the fact that capture rates of taxable income recorded for salaried workers, the self-employed and farmers are about 90, 60, and 40 percent, respectively. This issue is often a point of contention in Japan and it has been studied for many years. It has also been discussed whether a corporate enterprise tax system based on sales, salary, etc., and a taxpayer numbering system should be introduced; but this argument is yet to progress.

Woller (1999) claims that China's shrinking tax burden is partly due to nearly endemic level of tax evasion. "The Economist" (1989) points out that in 1989, the tax authorities of Shanghai seized the books of 10,361 private businessmen and found that 8,953 of them had evaded tax. As high as 86% of the 163,000 registered businessmen in Shanghai (3.2% of the city's workforce) may have evaded taxes, probably 100% of the unregistered ones did. According to Li (1995), tax evasion in China is estimated to have made the government a loss of around 100 billion Yuan a year.[2]

Fisman and Wei (2004) examine the relationship in China between the tariff schedule and the "evasion gap," which is defined as the difference between Hong Kong's reported exports to China at the product level and China's reported imports from Hong Kong. They have found that a one-percentage-point increase in the tariff rate is associated with 3 percent increase in evasion.

Due to such serious tax evasion, it is difficult for the China's government to capture the real economic activity, thus the part of economy does not get into the national accountings but becomes underground. Bajada and Schneider (2005) find that the size of China's underground economy

[2]Yuan is Chinese currency. One US dollar was about 6.82 Yuan in August 2008.

during 1991-1995 and 2000-2001 averaged 10.2% and 13.4% of the official GDP, respectively. Based on the report (2005) of the first census "China Economic Census 2004," National Bureau of Statistics of China adjusted the national accounting. For example, it is reported that both GDP and the service industry in 2004 were undervalued to 14.39% and 13.33% of GDP, respectively.[3]

An Institutional Innovation

According to Wan (2004), to capture a tax base, the Taiwan government has issued the lottery receipt, which is an official receipt printed with public lottery number (lottery receipt, also named as uniformed invoice with public lottery), as a means of tax collection since December 12, 1950. A lottery ticket number is incorporated into an official receipt, hence the lottery receipt is not only an official receipt but also a public lottery ticket simultaneously. Following Taiwan's experience, the Mainland China has also introduced this system in many provinces since 1998. Wan (2006, 2009b) reports that this system significantly raised the business tax revenue and its growth as well as total tax revenues based on panel data for 1998-2003 from a total 37 districts in Beijing and Tianjin. Based on a rich individual pseudo panel data set from six big cities in China, Wan (2009a) reports that the system markedly promoted the consumer's sales tax declaration by lottery receipt.

The Republic of Korea also "imported" this system from Taiwan in the

[3]GDP was re-estimated based on the information including the number of nationwide employees. In 2004, GDP was 15.99 trillion Yuan, but as much as 2.3 trillion Yuan was undervalued, of which 2.13 trillion Yuan was for the service industry.

1990s, and the new revised system seems to work well. The lottery receipt system has been revised to be a so called the "Korea Credit Card Tax Deduction System." If the consumer declares the total spending statement, for example, by credit card receipts, the government will give the consumer an income tax deduction based on the transaction volume stated in the receipts. This system is explained in Kim (2005). Armenia recently introduced the lottery receipt system.[4]

Berhan and Jenkins (2005) report that Northern Cyprus and Bolivia have introduced value-added tax refund system to refund some of tax paid on the purchases for which final consumer have obtained official receipts from the sellers, and that Indonesia has introduced the lottery receipt system just like Taiwan. According to the description in page 7 of Nihon Keizai Shimbun Morning Edition on June 3, 2006, the Philippines has also introduced the lottery receipt system since then.

Contributions and the Structure of this Paper

Until now the researches on tax evasion has been focused on the effects of governmental monitoring, punishment and consumer's attributes on the tax evaders. It is a institutional innovation that the government gives the taxpayers incentives to declare voluntarily the tax base by not inflicting punishment but giving a subsidy. As described in the above paragraphs, several governments have introduced the lottery receipt or cashback system, thus it would be desirable to clarify under what conditions the new system

[4] See the following WEB for the details. http://globalvoicesonline.org/2009/02/08/armenia-tax-lottery/

may work well.

We show that the subsidy works as an incentive mechanism which can mitigate the information asymmetry between the government and the taxpayer, and that the new system makes Pareto efficient taxation without evasion be practicable. This system may also contribute a lot to national system of accounting because the basic information on economic transaction will be sent to the government, and so, can be useful to mitigate the issue of underground economy in the world, especially in China, then contribute to fighting on corruption because the underground economic activity would produce a big space for the bureaucratic discretion.

The good economic performance during the transition period 1978-2008 has been called "the China Miracle." There are many studies on the reasons. For example, the "development strategy and economic reform" is emphasized in Lin et al. (1996). Based on the fact that there have been high saving-investment rates and high economic growth rates, Horioka and Wan (2007, 2008) explain why China saves so much. We would like to believe, just like the lottery receipt system, that there may also be many Pareto improving institutional innovations which have contributed to the economic growth, thus we should carefully analyze those effects of the apparently "special" systems on the economy in all fields in China.

The paper below is organized as follows. Section 2 contains a theoretical analysis based on a benchmark model. Section 3 contains the concluding remarks.

2 The Model

2.1 The Benchmark

An economic transaction is assumed to involve three types of agents: firms, consumers, and the government. The government wants to know the volume of transactions between firms and consumers in order to collect the tax, e.g., unit (or specific) tax. We assume that a large number of homogeneous firms, so large that the product market can be competitive, each seeking profit maximization within the competitive market. Also, that there are N homogeneous consumers; where $N > 1$.[5] When a consumer buys the product from a firm, the information on the purchased quantity is shared with the firm. The government cannot know about this sale unless it incurs a monitoring cost. Each pair of a consumer and a firm in a transaction have a free rider incentive not to pay taxes due to the government's inability to perfectly supervise the trading volumes between consumers and firms. It is very difficult for the government to monitor too many small firms, therefore the tax evasion problem is always serious. And because small firms are subject to always perfect competition, we consider a unit tax evasion issue in the perfectly competitive market.

Here we use the setting in Cremer and Gahvari (1993), and yet consider only one good for simplification. The firms' production technology is linear and the constant average and marginal cost is denoted by c. The output is subject to a per unit (or specific) tax of t and is sold at a consumer price of p.

[5] If N=1, the free rider problem will not arise.

The representative firm maximizes expected profits, π^e,

$$\pi^e = [p - c - g(1-\alpha) - (\alpha + (1-\alpha)\beta\tau)t]x. \tag{1}$$

Here, x denotes the firm's output. The magnitude α is the proportion reported by the tax-evading firm. The firm's taxes depend on its reported sales. The tax administration can confirm α without cost. The probability that the tax-evading firm caught by the tax administration is written as β $(0 \leq \beta \leq 1)$, while the penalty rate with an upper bound is denoted by τ $(0 \leq \tau \leq \bar{\tau})$. The social cost incurred by the tax-evading firm, simply called the cheating cost is denoted by a given function $g(1-\alpha)$, and convexly increases with the proportion $(1-\alpha)$ of sales unreported.

If $x > 0$, eq. (1) implies that the firm chooses α so that

$$q \equiv g(1-\alpha) + (\alpha + (1-\alpha)\beta\tau)t \tag{2}$$

is minimized. The first- and second-order conditions for this problem are

$$\frac{\partial q}{\partial \alpha} = -g'(1-\alpha) + (1-\beta\tau)t = 0, \tag{3}$$

$$g''(1-\alpha) > 0. \tag{4}$$

From eq. (3) it follows that a necessary condition to have an interior solution for α is

$$\beta\tau < 1. \tag{5}$$

We assume that the inequality (5) is satified, and $g(0) = 0,\ g(1) = \infty$. Then the solution will be in the interior. We put define

$$t^e \equiv (\alpha + (1-\alpha)\beta\tau)t \tag{6}$$

to denote the firm's expected tax payment per unit of output. The market equilibrium occurs at

$$p = c + g + t^e. \tag{7}$$

If $g + t^e < t$, the tax evasion problem arises. The reason is as follows. Suppose that a firm reports sales honestly, then the price of output will be $p^h = c + t$. Then $g + t^e < t$ makes $p < p^h$, the honest firm will disappear by market selection, and there are only tax-evading firms in the equilibrium.

The social cost incurred by auditing tax evasion, simply called the auditing cost is given by a function $d(\beta)$ which convexly increases with β. Following Cremer and Gahvari (1997), we assume that the government is benevolent, and that the government's problem is to maximize the utility, Λ, of a representative consumer with a reservation price v.

$$max_{\{t,\beta\}}\Lambda = v - p + ln(R), \tag{8}$$

where $p = c + g + t^e$, and $R \equiv t^e - d(\beta)$ is the net government revenue from per unit private goods. The logarithmic function $ln(R)$ is the utility from the public good. Cremer and Gahvari (1997) show that the government chooses the "social optimal," and obtains simulated solutions of $t^*(> 0),\ \beta^*(> 0)$, and $\tau^*(> 0)$ given that the firm optimally chooses $\alpha^*(1 > \alpha^* > 0)$, $g(\alpha^*) > 0$, $g(\alpha^*) + t^{e*} < t^*$. This is an equilibrium among the government, firms and consumers. Consequently, the honest firms are totally excluded by the cheating firms because of information asymmetry and the consumers' seeking for cheaper goods, and finally only the cheating firms exist in the market.

2.2 A Solution to Evasion

2.2.1 Cashback for Consumers as Incentives to Declare Tax

In the benchmark model, the positive $d(\beta^*)$ and $g(\alpha^*)$ are pure waste of social resources, thus the first best taxation, i.e. $\alpha^* = 1$, $g(\alpha^*) = 0$, $\beta^* = 0$, $d(\beta^*) = 0$, is not realized.

Here we consider the representative consumer's incentive to declare tax under a governmental subsidy or cashback. Assume that the government gives the consumer a subsidy s based on his or her declaration of purchase, to make him or her declare the tax, e.g. report the purchase to the government by asking for an official receipt from the firm, while keeping the auditing, $\beta^* > 0$ and $d(\beta^*) > 0$ as shown in the above section.[6]

Proposition 1. If $t^* - t^{e*} \geq s \geq t^* - t^{e*} - g(\alpha^*) > 0$, where the subsidy s is larger than and sufficiently close to $t^* - t^{e*} - g(\alpha^*)$ and $\alpha^* > 0$, this economy is Pareto improving.

Proof. To make the consumer buy the honest firm's goods (voluntarily and fully declare tax to the government), the consumer's incentive constraint

[6] In those countries where the lottery receipt system is used, the government monitors the tax evaders at the same time, while issuing lottery receipts. If the government does not audit, i.e. $\beta = 0$ and $d(\beta) = 0$, t^e may become zero.

is $p^s = c + t^* - s \leq p = c + t^{e*} + g(\alpha^*)$. *We obtain* $s \geq t^* - t^{e*} - g(\alpha^*)$. *And from the condition on evasion,* $p^h = c + t^* > p = c + t^{e*} + g(\alpha^*)$, $t^* - t^{e*} - g(\alpha^*) > 0$. *Therefore, we have* $s \geq t^* - t^{e*} - g(\alpha^*) > 0$. *The government's incentive constraint is* $s \leq t^* - t^{e*}$, *and it means that the additional increase of tax revenue from subsidy system has to cover the budget of the subsidy. If we set* $s = t^* - t^{e*} - g(\alpha^*) + \epsilon$, *and* $\epsilon \to 0^+$, *then the net increase of government revenue per unit output will be* $(t^* - t^{e*}) - (t^* - t^{e*} - g(\alpha^*) + \epsilon) = g(\alpha^*) - \epsilon > 0$ *provided that* $\alpha^* > 0$, *and the economy is Pareto improving.* Q.E.D.

2.2.2 Pareto Efficient Taxation without Evasion

After the consumer declares the tax to the government, the economy will shift to a new equilibrium by the subsidy without evasion. If t^* and $d(\beta^*)$ are kept unchanged, the net increase of government revenue will be $g(\alpha^*) - \epsilon > 0$. The government can change t^* to find a new social optimal tax t^{**}. In the new equilibrium, it is the best policy for the government to firstly seek for the minimum auditing cost. From eq. (3), let us assume that the minimum probability $\beta_{min} (0 \leq \beta_{min} < \beta^*)$ can assure an interior solution of α' for any t in an interval $[0,\ \bar{t}]$, then the government pays the lowest auditing cost $d(\beta_{min})$, and $0 \leq d(\beta_{min}) < d(\beta^*)$. The government has the following problem,

$$\text{(9)} \qquad max_{\{t,s\}} \Lambda = v - p + ln(R),$$

$$\text{(10)} \qquad s.t. \;\; p = c + t,$$

$$\text{(11)} \qquad p - s \leq p^e = c + g + t^e,$$

$$\text{(12)} \qquad R = t - s - d(\beta),$$

where eq. (11) is the consumer's incentive compatibility.

Proposition 2. The government can find an s^{**} to enable the practice of the Pareto efficient taxation t^{**} without evasion by the minimum auditing.

See Appendix for the proof. Specifically, we assume $g(1-\alpha) = 0.5(1-\alpha)^2$, and $0 \leq d(\beta) < 0.5$. By solving the above problem, we obtain the optimal solutions for tax and subsidy.

$$\text{(13)} \qquad t^{**} = (1-\beta\tau)^{-1}(1-\alpha^{**}),$$

$$\text{(14)} \qquad s^{**} = t^{**} - [\alpha^{**} + (1-\alpha^{**})\beta\tau]t^{**} - 0.5(1-\alpha^{**})^2,$$

where,

(15)

$$\alpha^{**} = -\frac{1-\beta\tau+\beta^2\tau^2}{1-\beta\tau} + \frac{\{(1-\beta\tau+\beta^2\tau^2)^2 + (1-\beta\tau)[(1-\beta\tau)(1-2d(\beta)) + 2\beta^2\tau^2]\}^{0.5}}{1-\beta\tau}.$$

In the equilibrium among the government, firms and consumers, the cheating firms are totally excluded by the consumers' tax declaration, and finally only the "honest" firms exist in the market. The "honest" firms are not honest by nature but passively so by the consumers' tax declaration, because they would still be eager to cheat if the consumers do not ask for an official receipt. The cheating cost is totally saved, and the auditing cost is partially saved and totally saved in the extreme case (sufficiently small auditing, discussed in the below section), thus the Pareto efficient taxation without evasion becomes practicable.

For the implementation of this subsidy system, the government needs detailed information on firms and consumers. Because the tax evasion repetitively occurs, and the government repetitively audits the taxpayers, it would not be difficult for the government to obtain infomation on the preferences of the related taxpayers.

2.2.3 An Example for Sufficiently Small Auditing

Next we consider how the firm behaves if the government makes $\beta_{min} \to 0^+$. The firm's best response may be to report nothing to the government, then $\alpha = 0$ and $t^e = 0$. But if $\beta_{min} \to 0^+$ assures the firm has an interior solution α^{***} $(0 < \alpha^{***} < 1)$ for $t \in [0,\ \bar{t}]$, then the government can find an s^{***} to enable the practice of the Pareto efficient taxation (t^{***}) without evasion and auditing. Certainly, this condition would hardly be realistic, because the firm reports positive α if and only if the firm meets the positive auditing, $\beta\tau > 0$. The audit cost $d(\beta_{min}) > 0$ is a threat to the cheating firms and makes $\alpha > 0$ and $t^e > 0$. This auditing cost is clearly the pure waste of social resource caused by both the information asymmetry and the taxpayers' free rider incentive, and by the government's inability of forcing all taxpayers to pay tax. We may call this type of cost "social cost of moral hazard." Because of this cost, the first-best taxation ($\beta^* = 0,\ d(\beta^*) = 0$) could hardly be realized.

Assume that the firm reports positive α when $\beta_{min} \to 0^+$, that is, $g'(1-\alpha) = (1-\beta\tau)t \to t$ of eq. (3) has an interior solution for α, then from eqs. (13)-(15) we obtain the following results,

$$\alpha^{***}_{\beta\to 0^+} = 2^{0.5} - 1 = 0.41, \tag{16}$$

$$t^{***}_{\beta\to 0^+} = 2 - 2^{0.5} = 0.59, \tag{17}$$

$$s^{***}_{\beta\to 0^+} = 3 - 2(2^{0.5}) = 0.17, \tag{18}$$

$$R^{***}_{\beta\to 0^+} = t^{**} - s^{**} = 2^{0.5} - 1 = 0.41, \tag{19}$$

and the ratio of s^{***} to t^{***} is,

$$\frac{s^{***}}{t^{***}} = 1 - \frac{2^{0.5}}{2} = 29.29\%. \tag{20}$$

Because we assume the consumer has linear preference on private goods, the above ratio of subsidy to tax is equivalent to the ratio of lottery prize to tax revenue in Wan (2006, 2009b), in which the ratio in nationwide China in 2002 was about 3.00%. Hence, in the case of China the ratio would be too low compared with the optimal value.

2.2.4 The Consumer's Reporting Cost

Next, we assume that per unit cost of consumer's reporting is ζ (> 0).

Proposition 3. If $g(\alpha^*) - \zeta > 0$ *and* $t^* - t^{e*} \geq s \geq t^* - t^{e*} - g(\alpha^*) + \zeta > 0$, *the government can Pareto improve the economy by giving the consumer a positive subsidy.*

Proof. *The consumer's incentive constraint is* $p^s = c + t^* + \zeta - s \leq p(= c + t^{e*} + g(\alpha^*))$. *And from the evasion condition* $t^* > t^{e*} - g(\alpha^*)$, *we obtain* $s \geq t^* - t^{e*} - g(\alpha^*) + \zeta > 0$. *The government's incentive constraint is* $s \leq t^* - t^{e*}$. *If we assume that* $s = t^* - t^{e*} - g(\alpha^*) + \zeta + \epsilon,\ \epsilon \to 0^+$, *the net increase of government revenue per unit output will be* $(t^* - t^{e*}) - (t^* - t^{e*} - g(\alpha^*) + \zeta + \epsilon) = g(\alpha^*) - \zeta - \epsilon > 0$ *provided that* $g(\alpha^*) - \zeta > \epsilon$. Q.E.D.

3 Conclusions

The literature on tax evasion has focused on the effects of governmental monitoring, punishment and consumers' attributes on the tax evaders. It is an institutional innovation that the government gives the taxpayers incentives to declare voluntarily the tax base by not inflicting punishment but giving a subsidy or cashback. This paper theoretically examines what conditions make this system work well, and shows that if the government gives the consumer a proper subsidy under a competitive firm's market, the consumer will voluntarily and fully declare unit tax so that the firm cannot cheat, and that the cheating and auditing costs can be saved, thus Pareto improving and efficient taxation without evasion becomes practicable. By this system, the government can prevent the tax evasion caused by collusion between consumers and firms and can collect unit taxes effectively.

For the implementation of this subsidy system in a real world, the government needs detailed information on firms and consumers. Because the tax evasion repetitively occurs, and the government repetitively audits the taxpayers, it would not be difficult for the government to obtain infomation on the preferences of the related taxpayers. And because we assume the consumer has linear preference on private goods, the ratio of subsidy to tax is equivalent to the ratio of lottery prize to tax revenue in Wan (2006, 2009b), in which the ratio in nationwide China in 2002 was about 3.00%. Hence, the China's government can raise the efficiency of the lottery receipt system by raising the ratio of prize to tax since the ratio in China has been too low compared with the optimal value.

We believe that the system will have a significant influence on future tax collection policies in the world. This system may also contribute a lot to national system of accounting because the basic information on economic transaction will be sent to the government, and so, can be useful to mitigate the issue of underground economy, then contribute to fighting on corruption because the underground economic activity would produce big space for the bureaucratic discretion.

For future research, we must clarify theoretically and more specifically the optimal policy, the consumer and the firm behaviors in imperfectly competitive markets, the non-benevolent government, political processes, and different purposes between the finance ministry and the tax bureau, etc. Furthermore, we must address not only the issue of efficiency but also the issue of interpersonal equity or fairness. Finally we must also empirically examine the effect of this new system on tax evasion based on macro and micro data sets from the countries except China where the system has been introduced.

Appendix

Proof of Proposition 2

First, we set the penalty payment τ to be its maximum, and the probability of finding tax evasion β to be its minimum value, and assume that the inequality (5), i.e. $\beta\tau < 1$, is satisfied. Then β and τ are parameters. From eqs. (10) and (11), we obtain

$$c + t - s \leq c + g + t^e, \tag{A.1}$$

and subtract c from both sides to have

$$t - s \leq g + t^e. \tag{A.2}$$

Assume that we find the minimum s to make $t - s = g + t^e$, and from eq. (12) we get the net tax revenue,

$$\begin{aligned} R &= t - s - d(\beta), \\ &= g + t^e - d(\beta). \end{aligned} \tag{A.4}$$

Next, to find the solution easily, we transform the problem shown in eqs. (9)-(12) into the following one with only one choice variable,

$$\begin{aligned} max_{\{t\}}\Lambda &= v - p + ln(R), \\ &= v - (c + t) + ln[g + t^e - d(\beta)]. \end{aligned} \tag{A.5}$$

The first order condition with respect to t is,

$$\frac{\partial \Lambda}{\partial t} = -1 + \frac{1}{g + t^e - d(\beta)}\left(\frac{\partial R}{\partial t}\right) = 0, \tag{A.6}$$

where,

$$\frac{\partial R}{\partial t} = \frac{\partial g}{\partial t} + \frac{\partial t^e}{\partial t} = \alpha + (1-\alpha)\beta\tau, \tag{A.7}$$

because we obtain the following relationships from eq. (3) and eq. (6),

$$\frac{\partial \alpha}{\partial t} = -\frac{1-\beta\tau}{g''}, \tag{A.8}$$

$$\frac{\partial g}{\partial t} = g'\left(-\frac{\partial \alpha}{\partial t}\right) = \frac{(1-\beta\tau)^2 t}{g''}, \tag{A.9}$$

$$\frac{\partial t^e}{\partial t} = [\alpha + (1-\alpha)\beta\tau] - \frac{(1-\beta\tau)^2 t}{g''}. \tag{A.10}$$

By rewriting eq. (A.6) with eq. (A.7) and eq. (6), we obtain

$$t = 1 - \frac{g(1-\alpha) - d(\beta)}{\alpha + (1-\alpha)\beta\tau}. \tag{A.11}$$

To obtain concrete solutions, we specify the function $g(1-\alpha)$,

$$g(1-\alpha) = 0.5(1-\alpha)^2, \tag{A.12}$$

$$g'(1-\alpha) = 1-\alpha, \tag{A.13}$$

$$g''(1-\alpha) = 1. \tag{A.14}$$

Recall the eq. (3), $g'(1-\alpha) = (1-\beta\tau)t$, then we solve α with eq. (A.13) and obtain

$$\alpha = 1 - (1-\beta\tau)t, \;\; or$$
$$t = \frac{1-\alpha}{1-\beta\tau}. \tag{A.15}$$

Then we substitute this t in eq. (A.11), and we obtain a equation for α,

$$\frac{1-\alpha}{1-\beta\tau} = 1 - \frac{0.5(1-\alpha)^2 - d(\beta)}{\alpha + (1-\alpha)\beta\tau}, \tag{A.16}$$

and this leads to

(A.17)
$$f(\alpha) \equiv (1-\beta\tau)\alpha^2 + (2 - 2\beta\tau + 2\beta^2\tau^2)\alpha - (1-\beta\tau)(1-2d(\beta)) - 2\beta^2\tau^2 = 0.$$

Corollary 1. If $0 \leq d(\beta) < 0.5$, *the solution of eq. (A.17) will be* $0 < \alpha^{**} < 1$.

Proof. Since $1-\beta\tau > 0$ and the intercept $-(1-\beta\tau)(1-2d(\beta)) - 2\beta^2\tau^2 < 0$ provided that $d(\beta) < 0.5$, there are two solutions for eq. (A.17), and the positive one is,

(A.18)
$$\alpha^{**} = -\frac{1-\beta\tau+\beta^2\tau^2}{1-\beta\tau} + \frac{\{(1-\beta\tau+\beta^2\tau^2)^2 + (1-\beta\tau)[(1-\beta\tau)(1-2d(\beta)) + 2\beta^2\tau^2]\}^{0.5}}{1-\beta\tau}.$$

If $f(1) > 0$, i.e.

$$d(\beta) > -1, \tag{A.19}$$

then we obtain $\alpha^{**} < 1$. The condition (A.19) is clearly satisfied. Q.E.D.

The optimal tax is solved by eqs. (A.15) and (A.18),

(A.19)

$$\begin{aligned} t^{**} =& (1-\beta\tau)^{-1}(1-\alpha^{**}), \\ =& \frac{(2-2\beta\tau+\beta^2\tau^2)}{(1-\beta\tau)^2} \\ & - \frac{\{(1-\beta\tau+\beta^2\tau^2)^2+(1-\beta\tau)[(1-\beta\tau)(1-2d(\beta))+2\beta^2\tau^2]\}^{0.5}}{(1-\beta\tau)^2}. \end{aligned}$$

Then the subsidy s is solved by eq. (A.2),

$$\begin{aligned} s^{**} &\geq t^{**} - t^{e**} - g^{**}, \\ &= t^{**} - [\alpha^{**} + (1-\alpha^{**})\beta\tau]t^{**} - 0.5(1-\alpha^{**})^2, \end{aligned} \tag{A.20}$$

and the net tax revenue is

$$\begin{aligned} R^{**} &= t^{**} - s^{**} - d(\beta), \\ &= g(1-\alpha^{**}) + t^{e**} - d(\beta), \\ &= 0.5(1-\alpha^{**})^2 + [\alpha^{**} + (1-\alpha^{**})\beta\tau]t^{**} - d(\beta). \end{aligned} \tag{A.21}$$

Q.E.D.

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