This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the IEEE Globecom 2011 proceedings.

An iterative semidefinite and Geometric programming technique for the SINR balancing in two-way relay network

G. Bournaka*†, K. Cumanan*, S.Lambotharan*and F. Lazarakis†
*Advanced Signal Processing Group, Department of Electronic and Electrical Engineering,
Loughborough University, UK.
Email:{G.Bournaka,K.Cumanan,S.Lambotharan}@lboro.ac.uk
†Institute of Informatics and Telecommunications, National Center for Scientific Research, Demokritos,
Athens, Greece.
Email: flaz@iit.demokritos.gr

***Abstract*—In this paper, we consider a two-way amplify-and-forward relaying scheme, which consists of multiple transceivers and *r* relay nodes. Assuming that both the transceivers and the relays are equipped with single antennas, we deploy a signal-to-interference and noise-ratio (SINR) balancing technique, where the smallest of the transceivers SINRs is maximized under a total transmit power constraint. We solve this problem through an iterative procedure that uses semidefinite and geometric programming along with bisection search methods. We evaluate the performance of the proposed scheme in terms of the mean SINR for various relays and power at the relays.**

I. Introduction

Reliability of wireless communications can be severely affected by signal fading, meaning that the signal attenuation can vary significantly over the course of a given transmission. Transmitting independent copies of the signal from different locations generates spatial diversity and can effectively combat fading. Cooperative communication generates this diversity due to spatial separation of the relaying terminals, overcoming the practical restrictions of deploying multiple antennas at the mobile terminals [1]. Moreover, relays enhance the link reliability between the source and destination, when there is a loss of the direct communication link between them.

Most of the results regarding distributed beamforming assume a one-way relaying scheme. The two-way relay was first introduced by Shannon [2], who derived bounds on the capacity region. Processing at relays can be regenerative e.g. decode-and-forward [3], compress-and-forward [4] or non regenerative e.g. amplify-and-forward, [5], [6] and [7]. Non regenerative relays are less complex and usually cause a smaller processing delay than regenerative ones. In addition, relays can be full-duplex or half-duplex. Full-duplex relays can transmit and receive at the same time and frequency but is hardly implementable. On the other hand, half-duplex relays, is realizable because they use orthogonal time or frequency. In this paper, we focus on nonregenerative, half-duplex two-way relays, because they have many advantages from an implementation viewpoint.

In general, relay transmission is an important technique to increase the capacity of the network and reduce the required transmit power. Recently, there has been increasingly gaining attention to the design of relays for multi-user wireless networks [8]. The wireless relay network, however, has a problem i.e. multiple access interference occurs since there are multiple transmission links in a network. With that motivation, a strategy should be applied that minimizes interference and at the same time improves the power efficiency of the relay system. Also, in order to maintain fairness among users, the resources should be allocated such that each user's SINR is balanced and maximised. The associated optimization problem is commonly referred to as "SINR Balancing" and aims to obtain optimal weighted vectors and transmission powers while maximising the worst-case user SINR.

Wireless nonregenerative relays under half-duplex constraints have been well studied in the literature. SINR balancing technique has been introduced in [9], where the principle of "downlink-uplink duality" has been applied to transform the original downlink problem into an equivalent uplink problem that is more easily to be solved. Another approach to solve this problem is to apply conic convex optimization e.g. [10]. Two-way AF relays have been also studied in [11] and [12]. However, the SNR balancing problem for two-way relay network has only been studied in [13], where one pair of terminals is considered. The solution is based on the assumption that the phase of the beamforming vector has to match the aggregated phase of the channel coefficients from the relay to the two transceivers.

In this paper, we consider the two-way, multi-user AF relay and aim to maximise the worst-case user SINR subject to the total transmit power budget. An iterative algorithm is being proposed that successively solves two different subproblems. The first one determines the relay coefficients under a sum power constraints at the relays to maximise the worst-case user SINR. The second one balances the SINR values of all users for a given set of relay coefficients by optimising transmission power at the sources. These two subproblems can be solved by applying a semi-definite relaxation approach together with

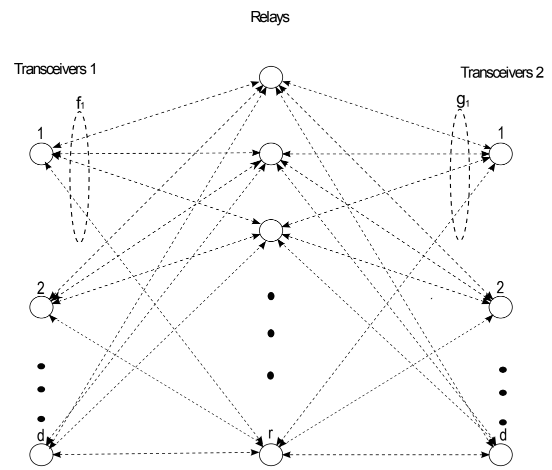

Fig. 1. A relay network of d pairs of nodes and r relay nodes, all single-antenna units.

a simple bisection search and geometric programming.

Notation: Lowercase letters are used for scalars. Vectors and matrices are denoted by boldface lowercase and uppercase letters respectively. We denote complex conjugate, transpose, and Hermitian transpose by $(\cdot)^*$, $(\cdot)^T$ and $(\cdot)^H$. We use $\mathrm{E}\{\cdot\}$ to denote statistical expectation, $\mathrm{Tr}\{\cdot\}$ and $\mathrm{Rank}(\cdot)$ represent the trace and rank of a matrix respectively. $\mathrm{diag}(\mathbf{m})$ denotes a diagonal matrix with the elements of the vector $\mathbf{m}$ as its diagonal entries. $\mathbf{M} \succeq 0$ means that $\mathbf{M}$ is a positive semidefinite matrix and $\odot$ stands for Schur-Hadamard (element-wise) multiplication.

II. System Model

Our model, as shown in Fig. 1, consists of r relays and d source-destination pairs. All nodes of the network are single-antenna units and can either transmit or receive information. That is, we consider a two-step, two-way, amplify-and-forward relaying scheme. In order to simplify our description, we will refer to the sources-destinations on the left side of the figure as Transceivers 1 and the sources-destinations on the right side of the figure as Transceivers 2. Let f_{ip}, represent the flat fading channel coefficients from the pth source of Transceivers 1 to the ith relay and g_{iq} represent the flat fading channel coefficients from the ith relay to the qth source of Transceivers 2 respectively. The $r \times 1$ complex vector $\mathbf{x}$ of the received signals at the relays can be written as:

$$\mathbf{x} = \sum_{p=1}^{d} \mathbf{f}_p s_p + \sum_{q=1}^{d} \mathbf{g}_q s_q + \boldsymbol{\nu} \tag{1}$$

where the following definitions are used: $\mathbf{x} \triangleq [x_1\ x_2\ \cdots\ x_r]^T$, $\boldsymbol{\nu} \triangleq [\nu_1\ \nu_2\ \cdots\ \nu_r]^T$, $\mathbf{f}_p \triangleq [f_{1p}\ f_{2p}\ \cdots\ f_{rp}]^T$, $\mathbf{g}_q \triangleq [g_{1q}\ g_{2q}\ \cdots\ g_{rq}]^T$, s_p, s_q are the information symbols transmitted by the pth and qth sources of Transceivers 1 and 2 respectively and $\boldsymbol{\nu}$ is the $r \times 1$ complex vector of the noise at the relays. The ith relay multiplies its received signal by a complex weight coefficient w_i^*. As a result, the vector of the signals transmitted by the relays is given by:

$$\mathbf{t} = \mathbf{W}\mathbf{x} \tag{2}$$

where $\mathbf{W} \triangleq \mathrm{diag}([w_1^*, w_2^*, \cdots, w_r^*])$. The received signals at the kth destination of Transceivers 1 and 2 are given by

$$y_{1k} = \mathbf{f}_k^T \mathbf{t} + n_k = \mathbf{f}_k^T \mathbf{W}\Big(\sum_{p=1}^{d} \mathbf{f}_p s_p + \sum_{q=1}^{d} \mathbf{g}_q s_q + \boldsymbol{\nu}\Big) + n_{1k}$$

$$y_{2k} = \mathbf{g}_k^T \mathbf{t} + n_k = \mathbf{g}_k^T \mathbf{W}\Big(\sum_{p=1}^{d} \mathbf{f}_p s_p + \sum_{q=1}^{d} \mathbf{g}_q s_q + \boldsymbol{\nu}\Big) + n_{2k} \tag{3}$$

n_{1k}, n_{2k} is the receiver noises at the kth transceiver for Transceivers 1 and 2 respectively. Since $\mathbf{a}^T \mathrm{diag}(\mathbf{b}) = \mathbf{b}^T \mathrm{diag}(\mathbf{a})$, we can rewrite (3) as:

$$\begin{aligned} y_{1k} \;=\;& \mathbf{w}^H \mathbf{F}_k \mathbf{g}_k s_{2k} + \mathbf{w}^H \mathbf{F}_k \sum_{q=1, q\neq k}^{d} \mathbf{g}_q s_q + \mathbf{w}^H \mathbf{F}_k \mathbf{f}_k s_{1k} \\ &+ \mathbf{w}^H \mathbf{F}_k \sum_{p=1, p\neq k}^{d} \mathbf{f}_p s_p + \mathbf{w}^H \mathbf{F}_k \boldsymbol{\nu} + n_{1k} \end{aligned} \tag{4}$$

$$\begin{aligned} y_{2k} \;=\;& \mathbf{w}^H \mathbf{G}_k \mathbf{f}_k s_{1k} + \mathbf{w}^H \mathbf{G}_k \sum_{p=1, p\neq k}^{d} \mathbf{f}_p s_p + \mathbf{w}^H \mathbf{G}_k \mathbf{g}_k s_{2k} \\ &+ \mathbf{w}^H \mathbf{G}_k \sum_{q=1, q\neq k}^{d} \mathbf{g}_q s_q + \mathbf{w}^H \mathbf{G}_k \boldsymbol{\nu} + n_{2k} \end{aligned} \tag{5}$$

where $\mathbf{F}_k \triangleq \mathrm{diag}(\mathbf{f}_k)$, $\mathbf{G}_k \triangleq \mathrm{diag}(\mathbf{g}_k)$. The third term in (4) depends on the signal s_k transmitted by transceiver k during the first time slot. As $\mathbf{F}_k \mathbf{f}_k s_k$ is known at transceiver k, the third term in (4) can be subtracted from $\mathbf{y}_{1k}$ and the residual signal can be processed at the kth transceiver to extract the symbol s_k. Similarly, the second term in (5) can be subtracted from $\mathbf{y}_{2k}$ to extract the symbol s_k. That is, the residual signals $\widetilde{y}_{1k} \triangleq y_{1k} - \mathbf{w}^H \mathbf{F}_k \mathbf{f}_k s_k$ and $\widetilde{y}_{2k} \triangleq y_{2k} - \mathbf{w}^H \mathbf{G}_k \mathbf{g}_k s_k$ are expressed as:

$$\begin{aligned} \widetilde{y}_{1k} \;=\;& \mathbf{w}^H \mathbf{F}_k \mathbf{g}_k s_{2k} + \mathbf{w}^H \mathbf{F}_k \sum_{q=1, q\neq 1}^{d} \mathbf{g}_q s_q \\ &+ \mathbf{w}^H \mathbf{F}_k \sum_{p=1, p\neq 1}^{d} \mathbf{f}_p s_p + \mathbf{w}^H \mathbf{F}_k \boldsymbol{\nu} + n_{1k} \end{aligned} \tag{6}$$

$$\begin{aligned} \widetilde{y}_{2k} \;=\;& \mathbf{w}^H \mathbf{G}_k \mathbf{f}_k s_{1k} + \mathbf{w}^H \mathbf{G}_k \sum_{p=1, p\neq 1}^{d} \mathbf{f}_p s_p + \\ &+ \mathbf{w}^H \mathbf{G}_k \sum_{q=1, q\neq 1}^{d} \mathbf{g}_q s_q + \mathbf{w}^H \mathbf{G}_k \boldsymbol{\nu} + n_{2k} \end{aligned} \tag{7}$$

and can be used at the corresponding transceivers to extract the desired information symbols.

In general, throughout the paper we use the following assumptions:

- $A1:$ The relay noises are assumed to be spatially white.
- $A2:$ The transmission power of the pth and qth source is $E\{|s_p|^2\} = P_p$ for $p = 1, \cdots, d$.
- $A3:$ The information symbols transmitted by the different sources are uncorrelated,i.e., $E\{s_p s_q^*\} = P_p \delta_{pq}$.
- $A4:$ The relay noise $\boldsymbol{\nu}$, the destination noises $\mathbf{n}_{1k}$ and $\mathbf{n}_{2k}$ and the information symbols $\{s_p\}_{p=1}^{d}$, $\{s_q\}_{q=1}^{d}$ are statistically independent.
- $A5:$ The instantaneous channel state information (CSI) in terms of the

channel vectors $\mathbf{f}_p$ and $\mathbf{g}_q$ is available at the processing nodes.

III. SINR Balancing

Our goal is to obtain the optimal powers P_{1k}, P_{2k} of the Transceivers 1 and 2, as well as the relay weight vector $\mathbf{w}$ through maximising the worst-case user SINR under a total power constraint. Mathematically, the optimization problem is formulated as:

$$\max_{\mathbf{p_1}\succeq 0,\mathbf{p_2}\succeq 0,\mathbf{w}} \min_k (SINR_{1k}, SINR_{2k}) \quad \text{s.t.} \quad \sum_{k=1}^{d} P_{1k} + \sum_{k=1}^{d} P_{2k} \le P_{dmax},\ P_R \le P_{Rmax} \tag{8}$$

$$\max_{\mathbf{p}\succeq 0,\mathbf{w}} \min_k (SINR_k) \quad \text{s.t.} \quad \sum_{k=1}^{d} P_k \le P_{dmax},\ P_R \le P_{Rmax} \tag{9}$$

where $SINR_{1k}$, $SINR_{2k}$, is the receive SINRs at the kth transceiver for Transceivers 1 and 2, P_R, P_{Rmax} is the actual and the given maximum transmit power at the relays and P_{dmax} is the given maximum total allowable transmit power at the users, $\mathbf{p_1} \triangleq [P_{11}\ P_{12}\cdots P_{1d}]$, $\mathbf{p_2} \triangleq [P_{21}\ P_{22}\cdots P_{2d}]$, and $\mathbf{p_1} \succeq 0$, $\mathbf{p_2} \succeq 0$ means that all entries of the vectors $\mathbf{p_1}$, $\mathbf{p_2}$ are non-negative.

Using (2) the total transmit power at the relays can be expressed as:

$$P_R = \mathrm{E}\{\mathbf{t}^H\mathbf{t}\} = \mathbf{w}^H\mathbf{D}\mathbf{w} \tag{10}$$

where $\mathbf{D} \triangleq \mathbf{E}\{\mathbf{X}^H\mathbf{X}\}$, $\mathbf{X} \triangleq \mathrm{diag}(\mathbf{x})$.

Using (1) the matrix $\mathbf{D}$ can be written as:

$$\mathbf{D} = \sum_{p=1}^{d} P_p \mathbf{F_p}\mathbf{F_p}^H + \sum_{q=1}^{d} P_q \mathbf{G_q}\mathbf{G_q}^H + \sigma^2\mathbf{I} \tag{11}$$

In order to derive the expressions for $SINR_{1k}$, $SINR_{2k}$, we calculate P_s^k, P_i^k, P_n^k, that represent the desired signal component power, the interference power and the noise power at the kth destination of the Transceivers 1 and 2 respectively. The kth desired signal power at the Transceivers 1 can be obtained as:

$$\begin{aligned} P_s^{1k} &= \mathrm{E}\{\mathbf{w}^H\mathbf{F}_k\mathbf{g}_k\mathbf{g}_k^H\mathbf{F}_k^H\mathbf{w}\}\mathrm{E}\{|s_{2k}|^2\} \\ &= P_{2k}\mathrm{E}\{\mathbf{w}^H(\mathbf{f}_k\odot\mathbf{g}_k)(\mathbf{f}_k^H\odot\mathbf{g}_k^H)\mathbf{w}\} \\ &= P_{2k}\mathbf{w}^H\mathrm{E}\{\mathbf{h}_k\mathbf{h}_k^H\}\mathbf{w} = P_{2k}\mathbf{w}^H\mathbf{R}_h^k\mathbf{w} \end{aligned} \tag{12}$$

and at the Transceivers 2 as

$$P_s^{2k} = P_{1k}\mathbf{w}^H\mathbf{R}_h^k\mathbf{w} \tag{13}$$

where $\mathbf{h}_k \triangleq (\mathbf{f}_k\odot\mathbf{g}_k) = [f_{1k}g_{1k}\ f_{2k}g_{2k}\cdots f_{Rk}g_{Rk}]^T$ and $\mathbf{R}_h^k \triangleq \mathrm{E}\{\mathbf{h}_k\mathbf{h}_k^H\}$. The vector $\mathbf{h}_k$ contains the total path coefficients between the kth source and its corresponding destination via different relays.

Denoting $D_k = \{1,2,\ldots,d\} - \{k\}$ and using (6), the interference power at the kth destination of Transceiver 1 is given by:

$$\begin{aligned} P_i^{1k} &= \mathrm{E}\left\{\mathbf{w}^H\mathbf{F}_k\left(\sum_{q\epsilon D_k} P_q\mathbf{g}_q\mathbf{g}_q^H + \sum_{p\epsilon D_k} P_p\mathbf{f}_p\mathbf{f}_p^H\right)\mathbf{F}_k^H\mathbf{w}\right\} \\ &= \mathrm{E}\left\{\mathbf{w}^H\left(\sum_{q\epsilon D_k} P_q(\mathbf{f}_k\odot\mathbf{g}_q)(\mathbf{f}_k^H\odot\mathbf{g}_q^H) + \sum_{p\epsilon D_k} P_p(\mathbf{f}_k\odot\mathbf{f}_p)(\mathbf{f}_k^H\odot\mathbf{f}_p^H)\right)\mathbf{w}\right\} \\ &= \mathbf{w}^H(\sum_{q\epsilon D_k} P_q\mathbf{Q}_{1k} + \sum_{p\epsilon D_k} P_p\mathbf{D}_{1k})\mathbf{w} = \mathbf{w}^H\mathbf{T_{1k}}\mathbf{w} \end{aligned} \tag{14}$$

where $\mathbf{h}_k^q \triangleq \mathbf{f}_k\odot\mathbf{g}_q$, $\mathbf{Q}_{1k} \triangleq \sum_{q\epsilon D_k}\mathbf{h}_k^q(\mathbf{h}_k^q)^H$ and $\mathbf{D}_{1k} \triangleq \sum_{p\epsilon D_k}(\mathbf{f}_k\odot\mathbf{f}_p)(\mathbf{f}_k^H\odot\mathbf{f}_p^H)$.

The interference power at the kth destination of Transceiver 2 is given by:

$$P_i^{2k} = \mathbf{w}^H(\sum_{p\epsilon D_k} P_p\mathbf{Q}_{2k} + \sum_{q\epsilon D_k} P_q\mathbf{D}_{2k})\mathbf{w} = \mathbf{w}^H\mathbf{T_{2k}}\mathbf{w} \tag{15}$$

where $\mathbf{h}_k^p \triangleq \mathbf{f}_k\odot\mathbf{g}_p$, $\mathbf{Q}_{2k} \triangleq \sum_{p\epsilon D_k}\mathbf{h}_k^p(\mathbf{h}_k^p)^H$ and $\mathbf{D}_{2k} \triangleq \sum_{q\epsilon D_k}(\mathbf{g}_k\odot\mathbf{g}_q)(\mathbf{g}_k^H\odot\mathbf{g}_q^H)$.

Using (6) and Assumption A4, we write the noise power at the kth destination of Transceiver 1 as:

$$\begin{aligned} P_n^{1k} &= \mathrm{E}\{\mathbf{w}^H\mathbf{F}_k\boldsymbol{\nu}\boldsymbol{\nu}^H\mathbf{F}_k^H\mathbf{w}\} + \sigma_n^2 \\ &= \mathbf{w}^H\mathbf{R}_f^k\mathbf{w} + \sigma_n^2 \end{aligned} \tag{16}$$

and Transceiver 2 as:

$$P_n^{2k} = \mathbf{w}^H\mathbf{R}_g^k\mathbf{w} + \sigma_n^2 \tag{17}$$

where $\mathbf{R}_f^k \triangleq \sigma_\nu^2\mathbf{F}_k\mathbf{F}_k^H$ and $\mathbf{R}_g^k \triangleq \sigma_\nu^2\mathbf{G}_k\mathbf{G}_k^H$.

Before using the above derivations to rewrite the optimization problem, we note that (9) is equivalent to two subproblems. The first one, calculates the relay weights $\{w_i\}_{i=1}^r$ and maximizes the worst case user SINR. Hence, it is formulated as:

$$\max_{\mathbf{w}} \min_k (SINR_{1k}, SINR_{2k}) \quad \text{s.t.} \quad P_R \le P_{Rmax}, \quad k = 1,\cdots,d \tag{18}$$

where P_{Rmax} is the maximum allowable total transmit power of the relays.

Using (11), (12), (13), (14), (15), (16), (17) and defining $\mathbf{W} \triangleq \mathbf{w}\mathbf{w}^H$, we rewrite the optimization problem in (18) as

$$\begin{aligned} &\max_{\mathbf{W}} \min_k \left(\frac{\mathrm{Tr}(P_{2k}\mathbf{R}_h^k\mathbf{W})}{\mathrm{Tr}((\mathbf{T}_{1k}+\mathbf{R}_f^k)\mathbf{W})+\sigma_n^2}, \frac{\mathrm{Tr}(P_{1k}\mathbf{R}_h^k\mathbf{W})}{\mathrm{Tr}((\mathbf{T}_{2k}+\mathbf{R}_g^k)\mathbf{W})+\sigma_n^2}\right) \\ &\quad \text{s.t. } \mathrm{Tr}(\mathbf{D}\mathbf{W}) \le P_{Rmax} \quad \text{for} \quad k = 1,2,\cdots,d \\ &\quad \mathrm{Rank}(\mathbf{W}) = 1,\ \mathbf{W} \succeq 0 \end{aligned} \tag{19}$$

The rank constraint in (19) is not convex. Using a semidefinite relaxation [14], we aim to solve the following optimization problem:

$$\begin{aligned} &\max_{\mathbf{W},t} \ t \\ &\text{s.t. } \mathrm{Tr}(\mathbf{W}(P_{2k}\mathbf{R}_h^k - t(\mathbf{T}_{1k}+\mathbf{R}_f^k))) \geq \sigma_n^2 t, \\ &\quad\ \mathrm{Tr}(\mathbf{W}(P_{1k}\mathbf{R}_h^k - t(\mathbf{T}_{2k}+\mathbf{R}_g^k))) \geq \sigma_n^2 t, \\ &\quad\ \mathrm{Tr}(\mathbf{DW}) \leq P_{Rmax} \ \text{ for } \ k=1,2,\cdots,d \end{aligned} \tag{20}$$

Due to the relaxation, the matrix $\mathbf{W}^*$ obtained by solving the optimization problem in (20) will not be of rank one all the time. If $\mathbf{W}^*$ happens to be rank one, then its principal eigenvector yields the optimal solution to the original problem. Otherwise, we have to use alternative techniques such as randomization techniques (e.g. [15]–[17]), to obtain a suboptimal rank-one solution from $\mathbf{W}^*$. The outage probability of obtaining higher than rank-one solution at varying users is summarized at the Table I. The simulation parameters for this Table are mentioned at the Simulations subsection.

For any fixed value of t the set of feasible W in (20) is convex. It follows that the optimization problem in (20) is quasi convex [18]. To solve (20), the following observation has to been used. Let t_{max} be the maximum value of t that is obtained by solving the optimization problem (20). If for any given t, the convex feasibility problem

TABLE I
TABLE OF OUTAGE PROBABILITY (OP) AT VARYING USERS

Users:	4	6	8	10
OP	:0.005	0.11	0.23	0.4

$$\begin{aligned} &\text{find } \ \mathbf{W} \\ &\text{s.t. } \mathrm{Tr}(\mathbf{W}(P_{2k}\mathbf{R}_h^k - t(\mathbf{T}_{1k}+\mathbf{R}_f^k))) \geq \sigma_n^2 t, \\ &\quad\ \mathrm{Tr}(\mathbf{W}(P_{1k}\mathbf{R}_h^k - t(\mathbf{T}_{2k}+\mathbf{R}_g^k))) \geq \sigma_n^2 t, \\ &\quad\ \mathrm{Tr}(\mathbf{DW}) \leq P_{Rmax} \ \text{ for } \ k=1,2,\cdots,d \quad \mathbf{W} \succeq 0 \end{aligned} \tag{21}$$

is feasible, then $t_{max} \geq t$. Conversely, if (21) is not feasible, then $t_{max} \leq t$. Based on this observation, one can check whether the optimal value t_{max} of the quasi-convex problem (20) is smaller or greater than any given value t using a bisection technique and solving a convex feasibility problem at each step. We start with a preselected interval $[l, u]$, that contains the optimal value t_{max}. We then solve the convex feasibility problem at the midpoint $t = (l+u)/2$, to determine whether the optimal value is larger or smaller than t. If (21) is feasible for this value of t, then we set $l = t$, otherwise, we choose $u = t$ and solve the convex feasibility problem in (21) again. This procedure is repeated until the difference between u and l is smaller than some preselected threshold δ.

We should note that the optimization in (20) maximises the worst case user SINR. If the number of relays is substantially higher than the number of users, all users SINR will tend to be equal to the worst case user SINR t_{max}. However, in general, this optimization at the relay will not be able to ensure SINR balancing. This is because, unlike transmit beamforming techniques [9], the relay transceiver has the inability to control the power usage for each user separately at the relays. Therefore, in order to balance the SINR of all users, we need to control the transmission power at the source level using the following Geometric programming approach.

We consider the optimization of SINR with regards to the power at the transmitters P_{1k}, P_{2k}, when the relay coefficient vector $\mathbf{w}$ is fixed. In this case, the following optimization problem has to be solved:

$$\begin{aligned} &\max_{\mathbf{p_1}\succeq 0, \mathbf{p_2}\succeq 0} \ \min_k (SINR_{1k}, SINR_{2k}) \\ &\text{s.t.} \quad \sum_{k=1}^{d} P_{1k} + \sum_{k=1}^{d} P_{2k} \leq P_{dmax} \end{aligned} \tag{22}$$

where P_{dmax} is the maximum allowable total transmit power at the transmitters.

Using (12), (13), (14), (15), (16), (17) and assuming that the relay weights $\{w_i\}_{i=1}^r$ are fixed, the optimization problem in (22) becomes:

$$\begin{aligned} &\max_{\mathbf{p_1}\succeq 0} \min_k \left(\frac{P_{2k}\mathbf{G}_h^k}{\sum_{q\epsilon D_k} P_q \mathbf{E}_h^{1k} + \sum_{p\epsilon D_k} P_p \mathbf{F}_h^{1k} + \mathbf{N}_{1k}} \right) \\ &\max_{\mathbf{p_2}\succeq 0} \min_k \left(\frac{P_{1k}\mathbf{G}_h^k}{\sum_{p\epsilon D_k} P_p \mathbf{E}_h^{2k} + \sum_{q\epsilon D_k} P_q \mathbf{F}_h^{2k} + \mathbf{N}_{2k}} \right) \\ &\text{s.t.} \quad \sum_{k=1}^{d} P_{1k} + \sum_{k=1}^{d} P_{2k} \leq P_{dmax} \end{aligned} \tag{23}$$

where $\mathbf{G}_h^k \triangleq \mathrm{Tr}(\mathbf{R}_h^k\mathbf{W})$, $\mathbf{E}_h^{1k} \triangleq \mathrm{Tr}(\mathbf{Q}_{1k}\mathbf{W})$, $\mathbf{E}_h^{2k} \triangleq \mathrm{Tr}(\mathbf{Q}_{2k}\mathbf{W})$, $\mathbf{F}_h^{1k} \triangleq \mathrm{Tr}(\mathbf{D}_{1k}\mathbf{W})$, $\mathbf{F}_h^{2k} \triangleq \mathrm{Tr}(\mathbf{D}_{2k}\mathbf{W})$, $\mathbf{N}_{1k} \triangleq \mathrm{Tr}(\mathbf{R}_f^k\mathbf{W})$ and $\mathbf{N}_{2k} \triangleq \mathrm{Tr}(\mathbf{R}_g^k\mathbf{W})$.
The above problem can be rewritten as:

$$\begin{aligned} &\max_{\mathbf{p_1},\mathbf{p_2},\widetilde{t}} \ \widetilde{t} \\ &s.t. \ \sum_{k=1}^{d} P_{1k} + \sum_{k=1}^{d} P_{2k} \leq P_{dmax}. \\ &\mathbf{E}_h^{1k} \sum_{q\epsilon D_k} P_q P_{2k}^{-1}\widetilde{t} + \mathbf{F}_h^{1k} \sum_{p\epsilon D_k} P_p P_{2k}^{-1}\widetilde{t} + \mathbf{N}_{1k} P_{2k}^{-1}\widetilde{t} < \mathbf{G}_h^k \\ &\mathbf{E}_h^{2k} \sum_{p\epsilon D_k} P_p P_{1k}^{-1}\widetilde{t} + \mathbf{F}_h^{2k} \sum_{q\epsilon D_k} P_q P_{1k}^{-1}\widetilde{t} + \mathbf{N}_{2k} P_{1k}^{-1}\widetilde{t} < \mathbf{G}_h^k \end{aligned} \tag{24}$$

The problem (24) is convex and belongs to the class of geometric programming [19] . As a result, it can be efficiently solved using numerical methods such as cvx [18].
The iterative SINR-balancing algorithm is summarized in Table II.

IV. SIMULATIONS

We consider two numerical examples, where a network of 2 source-destination pairs is assumed. The noise power at the relays and at the receivers is assumed to be equal to 0.1. The initial signal power at each source is equal to 1 and the maximum allowable sum power at the receivers

This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the IEEE Globecom 2011 proceedings.

TABLE II
ALGORITHMIC SOLUTION OF THE SINR BALANCING PROBLEM

1:**Initialize** $\mathbf{P}_1^{(0)} = [1, ..., 1]^T$, $\mathbf{P}_2^{(0)} = [1, ..., 1]^T$.
2:**Repeat**
3:Solve (21) with fixed $\mathbf{P}_{1d}$, $\mathbf{P}_{2d}$ to obtain an updated $\mathbf{W}$.
4:Solve (24) with fixed $\mathbf{W}$, to obtain an updated $\mathbf{P}_{1d}$, $\mathbf{P}_{2d}$.
5:**Until** the SINR converges.

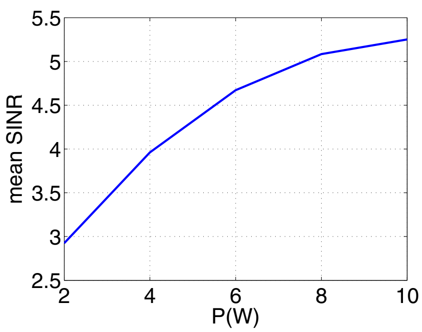

Fig. 2. The mean SINR versus the maximum allowable relay transmit power P_{Rmax} at the relays for a network with 5 relays.

is set to $P_{dmax} = 4$. In Fig. 2 the maximum allowable transmit power at the relays versus the mean SINR is plotted for a network with 5 relays. From this figure, we see that the performance of the users is improved as the maximum allowable transmit power at the relays is increased. In Fig. 3 the mean SINR versus different number of relays is plotted, while the maximum allowable transmit power at the relays is fixed to $P_{max} = 5$. Again, it can be seen that the performance at the receivers is substantially improved when the number of the relays is increased. Finally, for producing the results of Table I a network of 5 relays with $P_{rmax} = 5$ is assumed.

V. CONCLUSION

In this paper, we studied the problem of SINR balancing in a multiuser two-way relay network which consists of r relay nodes and d transceiver pairs. We considered two subproblems to achieve balanced SINRs over all transceivers. At the first subproblem, we designed the relay coefficients through maximizing the worst case user SINR subject to a constraint that guarantees a minimum transmission power at the relays. We observed that this approach does not guarantee equal SINR over all transceivers, unless the number of the relays is substantially bigger than the number of users. With a geometric programming approach, we have herein shown that by controlling the power of each user, we are able to provide a balanced SINR over all transceivers. Simulation results validate the efficiency of the algorithm.

ACKNOWLEDGMENT

This work has been supported by a Joined Studenship between the Center for Scientific Research Demokritos and Loughborough University, as well as by the EPSRC of UK under Grants E041817 and G020442.

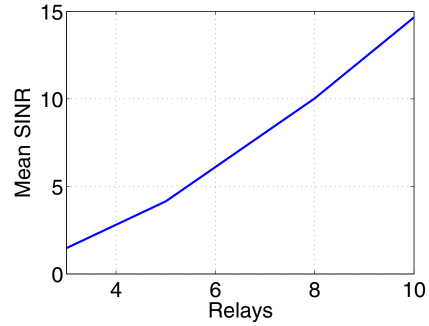

Fig. 3. The mean SINR versus different number of relays for a network with maximum allowable transmit power P_{Rmax}=5

REFERENCES

[1] Y. W. Hong, W. J. Huang, F. H. Chiu, and C. C. J. Kuo, "Cooperative communications in resource-constrained wireless networks," *IEEE Signal Process. Mag.*, no. 3, pp. 47–57, May. 2007.
[2] C. E. Shannon, "Two-way communication channels," in *Proc. 4th Berkeley Symp. Math. Stat. Prob*, pp. 351–384, 1961.
[3] Y. Jia and A. Vosoughi, "Two-way relaying for energy constrained systems: Joint transmit power optimization," in *Proc. IEEE Inter. Conf. Accous., Speech and Sign.ICASSP*, pp. 3026 – 3029, Jun. 2010.
[4] G. Kramer, M. Gastpar, and G. Gupta, "Cooperative strategies and capacity theorems for relay networks," *IEEE Trans. Inform. Theory*, vol. 51, pp. 3037– 3063, Sep. 2005.
[5] R. Krishna, K. Cumanan, Z. Xiong, and S. Lambotharan, "A novel cooperative relaying strategy for wireless networks with signal quantization," *IEEE Trans. Vech. Tech.*, vol. 59, no. 1, pp. 458–489, Jan. 2010.
[6] J. N. Laneman, D. N. C. Tse, and G. W. Wornell, "Cooperative diversity in wireless networks: Efficient protocols and outage behavior," *IEEE Trans. Inform. Theory*, vol. 50, no. 12, pp. 3062–3080, Dec. 2004.
[7] R. U. Nabar, H. Bolcskei, and F. W. Kneubuhler, "Fading relay channels: Performance limits and space-time signal design," *IEEE J. Sel. Areas Commun.*, vol. 22, pp. 1099–1109, Aug. 2004.
[8] V. Fazeli-Dehkordy, S. Shahbazpanahi, and S. Gazor, "Multiple peer-to-peer communications using a network of relays," *IEEE Trans. Signal Process.*, vol. 57, pp. 3053– 3062, Aug. 2009.
[9] M. Schubert and H. Boche, "Solution of the multiuser downlink beamforming problem with individual SINR constraints," *IEEE Trans. Vehicular Technol.*, vol. 53, no. 1, pp. 18– 28, Jan. 2004.
[10] A. Wiesel, Y. C. Eldar, and S. Shamai, "Linear precoding via conic optimization for fixed MIMO receivers," *IEEE Trans. Signal Process.*, vol. 54, no. 1, pp. 161– 176, Jan. 2006.
[11] R. Zhang, Y.-C. Liang, C. C. Chai, and S. Cui, "Optimal beamforming for two-way multi-antenna relay channel with analogue network coding," *IEEE J. Sel. Areas Commun.*, vol. 27, no. 5, pp. 699 – 712, Jun. 2009.
[12] J. Joung and A. H. Sayed, "Multiuser two-way amplify-and-forward relay processing and power control methods for beamforming systems," *IEEE Trans. Signal Process.*, vol. 58, pp. 1833 – 1846, Mar. 2010.
[13] V. Havary-Nassab, S. Shahbazpanahi, and A. Grami, "Optimal distributed beamforming for two-way relay networks," *IEEE Trans. Signal Process.*, vol. 58, pp. 1238 – 1250, Mar. 2010.
[14] M. Bengtsson and B. Ottersten, *Handbook of Antennas in Wireless Communications*. Optimal and Suboptimal Transmit Beamforming: Boca Raton, FL:CRC, Aug. 2001.
[15] N. Sidiropoulos, T. Davidson, and Z. Q. Luo, "Transmit beamforming for physical-layer multicasting," *IEEE Trans. Sign. Process.*, vol. 54, no. 6, pp. 2239 – 2251, Jun. 2006.
[16] P. Tseng, "Further results on approximating nonconvex quadratic optimization by semidefinite programming relaxation," *SIAM J. Optim.*, vol. 14, no. 1, pp. 268– 283, Jul. 2003.
[17] S. Zhang, "Quadratic maximization and semidefinite relaxation," *Math. Program.*, vol. 87, pp. 453–465, 2000.
[18] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge, UK: Cambridge University Press, 2004.
[19] M. Chiang, *Geometric Programming for Communication Systems*. now Publishers Inc., 2005.