

# TimeRank: a Random Walk approach for Community Discovery in Dynamic Networks

---

Ilias Sarantopoulos<sup>1,2</sup>   Dimitrios Papatheodorou<sup>2,4</sup>   Dimitrios Vogiatzis<sup>1,3</sup>  
Grigorios Tzortzis<sup>1</sup>   Georgios Paliouras<sup>1</sup>

<sup>1</sup>National Centre for Scientific Research (NCSR) “Demokritos”, Athens, Greece,

<sup>2</sup>Athens University of Economics and Business, Athens, Greece

<sup>3</sup>The American College of Greece, Athens, Greece,

<sup>4</sup>Aalto University, Espoo, Finland

# Table of contents

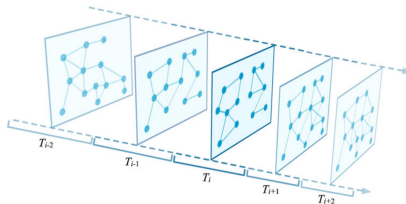
1. Introduction
2. Background
3. TimeRank
4. Datasets & Experiments
5. Future Work

# Introduction

---

# The setting

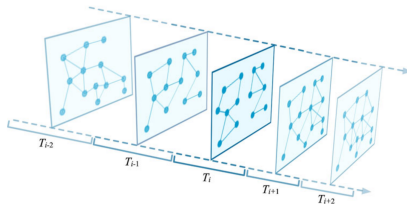
- An undirected graph which represents a (social) network



**Figure 1:** An example of a dynamic network consisting of five timeframes as shown in [1]

# The setting

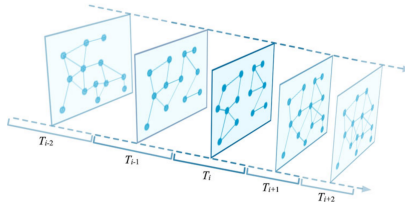
- An undirected graph which represents a (social) network
- Graph topology changes over time



**Figure 1:** An example of a dynamic network consisting of five timeframes as shown in [1]

# The setting

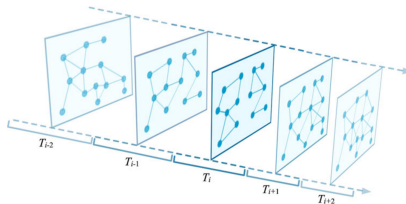
- An undirected graph which represents a (social) network
- Graph topology changes over time
- We use several discrete snapshots of the network and refer to them as timeframes



**Figure 1:** An example of a dynamic network consisting of five timeframes as shown in [1]

# Dynamic Community Finding

- The communities (i.e. Clusters) in each timeframe (**Detection**)
- Track the communities across time (**Tracking**)



**Figure 2:** An example of a dynamic network consisting of five timeframes as shown in [1]

## Two Step methods

1. 1st Step: Detect communities in each Timeframe graph (louvain, spectral etc.)
2. 2nd Step: Match communities between timeframes based on some similarity measure

## One Step methods

Perform detection and tracking in one step (e.g. using Non-Negative Tensor Factorisation)



# Background

---

# MutuRank - Purpose

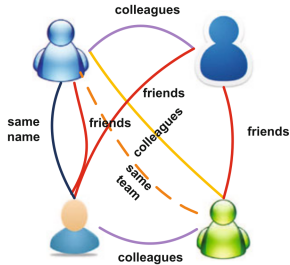


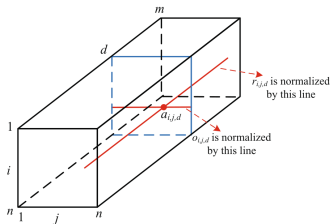
Figure 3: A multi relational network as described in [5]

- Perform Random walk on edges and relations
- Rank nodes and relations
- Transform Multi-relational network to Single relational
- Detect communities

# MutuRank Algorithm

$$p_i^t = \alpha \sum_{j=1}^n \sum_{d=1}^m p_j^{t-1} \cdot o_{i,j,d} \cdot \text{Prob}^{t-1}[d|j] + (1 - \alpha)p_i^*, \quad (1)$$

$$q_d^t = \beta \sum_{i=1}^n \sum_{j=1}^n p_j^{t-1} \cdot r_{i,j,d} \cdot \text{Prob}^{t-1}[i|j] + (1 - \beta)q_d^* \quad (2)$$



**Figure 4:** Depiction of the normalisation of tensors  $\mathcal{O}$  and  $\mathcal{R}$  in MutuRank and TimeRank (source [5]).

Relations and nodes yield mutual influence.

Use  $\mathbf{q}$  to transform to Single Relational Network (SRN)

$$w_{i,j} = \sum_{d=1}^m q_d \cdot a_{i,j,d}, \quad (3)$$

Then, detect communities using any algorithm

# TimeRank

---

# Adapting Muturank for community tracking

- Although communities evolve, they tend to have common structure.

# Adapting Muturank for community tracking

- Although communities evolve, they tend to have common structure.
- Nodes and Communities in different timeframes are disconnected

# Adapting Muturank for community tracking

- Although communities evolve, they tend to have common structure.
- Nodes and Communities in different timeframes are disconnected
- Most Community detection algorithms rely on close connectivity



# Adapting Muturank for community tracking

$N$  = # of Nodes

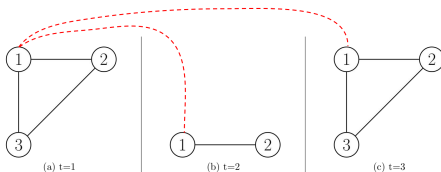
$T$  = # of Timeframes

- Relations (Muturank) = Timeframes (Timerank)
- Adapt muturank representation to include time-varying nodes:  
 $(N \times T) \times (N \times T) \times T$
- Add intra-timeframe edges (network edges)
- Add inter-timeframe edges: connect the same node with its image between timeframes.
- Apply muturank  $\rightarrow$  temporal network with  $N \times T$  nodes
- Perform clustering on this network and extract dynamic communities

# Timerank - One to All Connection (AOC)

- 'One to All Connection' connects each node  $i_t$  with all its occurrences in other timeframes
- for each node  $i_t$  add the following pairs of edges in the network

$$\{(i_1, i_t), \dots, (i_{t-1}, i_t), (i_{t+1}, i_t), \dots, (i_T, i_t)\}$$

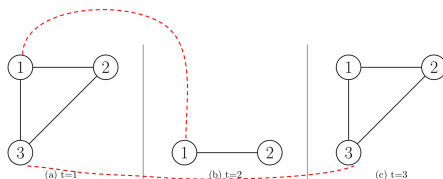


**Figure 5:** Sample dynamic network with 3 timeframes demonstrating AOC connections

# Timerank - Next Occurrence Connection (NOC)

- 'Next Occurrence Connection' connects each node  $i_t$  with its images in the previous and next timeframes in which this node exists
- for each node  $i_t$  add the following pairs of edges in the network

$$\{(i_{t-1}, i_t), (i_{t+1}, i_t)\}$$



**Figure 6:** Sample dynamic network with 3 timeframes demonstrating NOC connections

---

## Algorithm 1 TimeRank Algorithm

---

- 1: Create  $(N \times T)$  adjacency matrices for each timeframe
  - 2: Add inter-time edges
  - 3: Compose tensor  $\mathcal{A} \in \mathbb{R}^{(N \times T) \times (N \times T) \times (T)}$
  - 4: Apply MutuRank algorithm on  $\mathcal{A}$  and get ranking of timeframes
  - 5: Create time-weighted network with  $(N \times T)$  nodes
  - 6: Perform clustering on this network and extract dynamic communities
-

## Datasets & Experiments

---

Synthetic Datasets using Dynamic Benchmark Network Generator [3], which is based on [4].

## Expand/Contract events

1. 1000 nodes / 5 timeframes
2. 32 communities
3. 10 expand, 10 contract
4. 25% expansion/contraction rate

## Hide/Appear events

1. 1000 nodes / 5 timeframes
2. 32 communities
3. 10% of communities hide

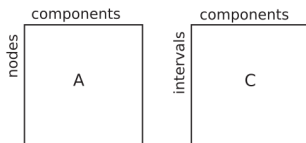
Two step approach : **Group Evolution Discovery (GED)**, Piotr Bródka, Stanislaw Saganowski, and Przemyslaw Kazienko

1. Run community detection algorithm on each timeframe (e.g. Louvain)
2. Match communities between sequential timeframes using a similarity measure

## Compared Methods (2/2)

One step approach : **Non-Negative Tensor Factorisation**, Laetitia Gauvin, André Panisson, and Ciro Cattuto

Perform PARAFAC decomposition on 3-way tensor  $T \in \mathbb{R}^{N \times N \times S}$ , where  $N$  is the number of nodes of the network and  $S$  the number of network snapshots.



**Figure 7:** Schematic representation of the factorisation result for an undirected temporal network from [2].



1. GED
  - Ground truth communities as input
2. NNTF
  - Random restarts for initialisation of factors **A**, **B** and **C**.
3. TimeRank
  - AOC connection - uniform distribution for  $q$
  - NOC connection - uniform distribution for  $q$
  - AOC connection - run Muturank
  - NOC connection - run Muturank

- NMI for overlapping clusters
- Omega index (rand index expansion for overlapping clusters)
- BCubed

# Results

|          | Expand/Contract |              |              |              |              | Hide/Appear  |              |              |               |              |
|----------|-----------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|---------------|--------------|
| Method   | NMI             | Omega        | Prec         | Rec          | F1           | NMI          | Omega        | Prec         | Rec           | F1           |
| TR-AOC-U | 0.866           | 0.874        | 0.905        | 0.882        | 0.893        | <b>1.000</b> | <b>1.000</b> | <b>1.000</b> | <b>1.000</b>  | <b>1.000</b> |
| TR-NOC-U | 0.908           | 0.919        | 0.944        | 0.921        | 0.933        | 0.880        | 0.890        | 0.912        | 0.963         | 0.937        |
| TR-AOC   | 0.849           | 0.864        | 0.890        | 0.883        | 0.886        | <b>1.000</b> | <b>1.000</b> | <b>1.000</b> | <b>1.0000</b> | <b>1.000</b> |
| TR-NOC   | <b>0.923</b>    | <b>0.954</b> | <b>0.964</b> | <b>0.944</b> | <b>0.953</b> | 0.910        | 0.918        | 0.935        | 0.963         | 0.949        |
| NNTF     | 0.805           | 0.8445       | 0.842        | 0.864        | 0.853        | <b>1.000</b> | <b>1.000</b> | <b>1.000</b> | <b>1.000</b>  | <b>1.000</b> |
| GED      | 0.464           | 0.659        | 0.924        | 0.572        | 0.707        | 0.531        | 0.700        | 0.901        | 0.662         | 0.763        |

**Table 1:** Tables for Expand/Contract and Hide/Appear Datasets

|        | Expand/Contract |       |       |       |       | Hide/Appear |       |       |       |       |
|--------|-----------------|-------|-------|-------|-------|-------------|-------|-------|-------|-------|
| Method | $t_1$           | $t_2$ | $t_3$ | $t_4$ | $t_5$ | $t_1$       | $t_2$ | $t_3$ | $t_4$ | $t_5$ |
| TR-AOC | 0.212           | 0.196 | 0.198 | 0.196 | 0.199 | 0.214       | 0.189 | 0.199 | 0.191 | 0.207 |
| TR-NOC | 0.217           | 0.189 | 0.191 | 0.196 | 0.207 | 0.218       | 0.183 | 0.193 | 0.193 | 0.213 |

**Table 2:** Values for  $q$  distribution for the *Expand/Contract* and *Hide/Appear* datasets

- Dynamic community : subreddit
- Timeframe: week
- Nodes: users
- Edges: replies

# Reddit Experiment 1

| Method   | 4 Timeframes |              |              |              |              | 8 Timeframes |              |              |              |              |
|----------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|
|          | NMI          | Omega        | Prec         | Rec          | F1           | NMI          | Omega        | Prec         | Rec          | F1           |
| TR-AOC-U | 0.385        | 0.316        | 0.557        | 0.56         | 0.558        | 0.319        | 0.243        | 0.547        | 0.543        | 0.545        |
| TR-NOC-U | 0.480        | 0.428        | 0.639        | 0.619        | 0.629        | 0.377        | 0.455        | 0.612        | 0.580        | 0.596        |
| TR-AOC   | 0.390        | 0.373        | 0.576        | 0.605        | 0.591        | 0.295        | 0.217        | 0.531        | 0.521        | 0.526        |
| TR-NOC   | 0.457        | 0.473        | 0.633        | 0.623        | 0.628        | <b>0.435</b> | <b>0.537</b> | 0.610        | <b>0.654</b> | <b>0.631</b> |
| NNTF     | 0.447        | 0.496        | 0.627        | <b>0.642</b> | 0.634        | 0.395        | 0.480        | 0.590        | <b>0.650</b> | 0.619        |
| GED-T    | <b>0.584</b> | <b>0.776</b> | <b>1.000</b> | 0.625        | <b>0.769</b> | 0.323        | 0.432        | <b>1.000</b> | 0.377        | 0.548        |

Table 3: Experiment 1 results for 4 and 8 timeframes

## Reddit Experiment 2

| Method   | 4 Timeframes |              |              |              |              | 8 Timeframes |              |              |              |              |
|----------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|
|          | NMI          | Omega        | Prec         | Rec          | F1           | NMI          | Omega        | Prec         | Rec          | F1           |
| TR-AOC-U | 0.050        | 0.068        | 0.480        | 0.544        | 0.510        | 0.028        | -0.007       | 0.438        | 0.438        | 0.438        |
| TR-NOC-U | 0.380        | 0.487        | 0.692        | <b>0.905</b> | 0.784        | 0.032        | 0.032        | 0.435        | 0.536        | 0.480        |
| TR-AOC   | 0.038        | 0.032        | 0.471        | 0.542        | 0.500        | 0.028        | -0.007       | 0.438        | 0.439        | 0.439        |
| TR-NOC   | <b>0.456</b> | <b>0.604</b> | 0.741        | <b>0.951</b> | <b>0.833</b> | 0.031        | 0.030        | 0.435        | 0.536        | 0.480        |
| NNTF     | 0.276        | 0.389        | 0.723        | 0.652        | 0.686        | <b>0.637</b> | <b>0.772</b> | 0.849        | <b>0.933</b> | <b>0.890</b> |
| GED-T    | 0.210        | 0.280        | <b>1.000</b> | 0.269        | 0.424        | 0.099        | 0.147        | <b>1.000</b> | 0.132        | 0.233        |

Table 4: Experiment 2 results for 4 and 8 timeframes

## Future Work

---

- Experiments on DBLP Data
- Mix Benchmark Data
- Use Multirank instead of Muturank
- Add Weights in inter-timeframe edges
- Scaling through parallelization



Questions?

# Evaluation measures

## BCubed

$$Precision(u, v) = \frac{Min(|T(u) \cap T(v)|, |C(u) \cap C(v)|)}{|C(u) \cap C(v)|}$$

$$Recall(u, v) = \frac{Min(|T(u) \cap T(v)|, |C(u) \cap C(v)|)}{|T(u) \cap T(v)|}$$

$$F_1 = 2 \cdot \frac{BCubed_{precision} \cdot BCubed_{Recall}}{BCubed_{precision} + BCubed_{Recall}}$$



P. Bródka, S. Saganowski, and P. Kazienko.

**Ged: the method for group evolution discovery in social networks.**

*Social Network Analysis and Mining*, 3(1):1–14, 2013.



L. Gauvin, A. Panisson, and C. Cattuto.

**Detecting the community structure and activity patterns of temporal networks: a non-negative tensor factorization approach.**

*PloS one*, 9(1):e86028, 2014.



D. Greene, D. Doyle, and P. Cunningham.

**Tracking the evolution of communities in dynamic social networks.**

*In Advances in social networks analysis and mining (ASONAM), 2010 international conference on*, pages 176–183. IEEE, 2010.



A. Lancichinetti, S. Fortunato, and F. Radicchi.

**Benchmark graphs for testing community detection algorithms.**

*Physical review E*, 78(4):046110, 2008.



Z. Wu, J. Cao, G. Zhu, W. Yin, A. Cuzzocrea, and J. Shi.

**Detecting overlapping communities in poly-relational networks.**

*World Wide Web*, 18(5):1373–1390, 2015.