Foundations — Formal Logics

Stasinos Konstantopoulos
NCSR 'Demokritos'

October 2023

The Dartmouth Summer Project

A Proposal for the

DARTMOUTH SUMMER RESEARCH PROJECT ON ARTIFICIAL INTELLIGENCE

June 17 - Aug. 16

We propose that a 2 month, 10 man study of artificial intelligence be carried out during the summer of 1956 at Dartmouth College in Hanover, New Hampshire. The study is to proceed on the basis of the conjecture that every aspect of learning or any other feature of intelligence can in principle be so precisely described that a machine can be made to simulate it. An attempt will be made to find how to make machines use language, form abstractions and concepts, solve kinds of problems now reserved for humans, and improve themselves. We think that a significant advance can be made in one or more of these problems if a carefully selected group of scientists work on it together for a summer.

Minsky and McCarthy's proposal to design general AI in two months, possibly an overpromise.

The Dartmouth Summer Project

Symbolic computation (McCarthy), connectionism (Minky and Rochester), and information theory (Shannon) joined forces to describe a framework for:

- manipulating words according to rules of reasoning and rules of conjecture [...] admitting a new word and rules whereby sentences containing it imply and are implied by others.
- arranging neurons so as to form concepts
- a theory of the complexity of functions
- self-improvement
- machine methods of forming abstractions from sensory and other data
- creativity: the difference between creative thinking and unimaginative competent thinking lies in the injection of a some randomness

Lecture Overview

In the final two lectures we will discuss:

- Computational logic, as the foundation for symbol manipulation
  - Syntactic restrictions: Dyadic logics, Horn logic, Modal logics
  - Semantics of equality, other functions over the elements of the domain besides equality
  - Semantics of the connectives: many-valued logics

Lecture Overview

In the final two lectures we will discuss:

- Computational logic, as the foundation for symbol manipulation
  - Syntactic restrictions: Dyadic logics, Horn logic, Modal logics
  - Semantics of equality, other functions over the elements of the domain besides equality
  - Semantics of the connectives: many-valued logics
- Computational learning theory, as the foundation for machine learning
  - The PAC framework
  - The theoretical assumptions underpinning statistical generalization

Useful Fragments of First Order Predicate Logic

Syntactic restrictions: Dyadic logics, Horn logic, Modal logics

Propositions

- A propositional variable is a symbol that represents the truth or falsehood of a proposition, a statement.
- Propositional variables have no arguments, i.e., have arity 0. When different entities share properties, the relevant propositions must be repeated and are not related in any formal way.

Examples

- p: Stasinos teaches 'KRR' at the AI MSc programme.
- p: Stasinos teaches 'Foundations' at the AI MSc programme.
- q: Angelos teaches 'Foundations' at the AI MSc programme.

Predicates

- A predicate has arguments. An interpretation of the predicate is a set of (tuples of) constants that fill the argumets.
- An interpretation that makes the predicate evaluate to True is a model of the predicate.

Examples

- P(X): X teaches 'KRR' at the AI MSc programme.
- Q(X): Stasinos teaches X at the AI MSc programme.
- R(X,Y): X teaches Y at the AI MSc programme.
- S(X,Y,Z): X teaches Y at the Z programme.

Complex Expressions

- So now we know the meaning, the *semantics* of a proposition or a predicate
  - i.e., we know how to decide if it is True or False.
- The logical connectives also have a semantics, so we can decide if a complex expression is True or False.
- The semantics of the logical connectives is an algebra.
  - You have already seen how Boole algebra is used to give a semantics to propositional and FOPL expressions.

Useful Fragments of First Order Predicate Logic

Dyadic Logics

Definition

The dyadic fragment of FOPL allows all FOPL statements that can be expressed with two variables.

Is dyadic

$$\mathrm{course}(Y) \wedge \mathrm{teaches}(X, Y) \wedge \mathrm{popular}(Y) \rightarrow$$
$$\mathrm{popular}(X) \vee \mathrm{hasPopularSubject}(Y)$$

Is not dyadic

$$\mathrm{teaches}(X, Y, From, To) \rightarrow \mathrm{activeInstructor}(X, From, To)$$

So, what's the point?

The dyadic fragment of FOPL allows reasoning about graphs and networks:

- Monadic predicates are node types
- Dyadic predicates are edge types

It is the theoretical infrastructure under RDF, Semantic Web technologies, and Knowledge Graphs; To be revisited in *KRR* and *Applications of AI*.

Useful Fragments of First Order Predicate Logic

Horn Logic

Definition

The Horn fragment of FOPL allows only one consequent.

Is Horn

$$\mathrm{teaches}(X, Y, \mathit{From}, \mathit{To}) \rightarrow \mathrm{activeInstructor}(X, \mathit{From}, \mathit{To})$$

Is not Horn

$$\mathrm{teaches}(X, Y) \wedge \mathrm{hasSubject}(Y, Z) \wedge \mathrm{popular}(Y) \rightarrow \mathrm{popular}(X) \vee \mathrm{popular}(Z)$$

So, what's the point?

Horn allows reasoning about tabular data:

- A ground predicate is a database table
- A defined predicate is a view formed by joining tables through variables shared between body literals.

It is the theoretical infrastructure under Prolog and deductive databases; To be revisited in *KRR* and *Applications of AI*.

Useful Fragments of First Order Predicate Logic

Modal Logics

Adding Operators

$$\alpha \leftarrow p, q, , ..., \mathsf{T}, \mathsf{F}$$

$$\phi \leftarrow \alpha | \neg\phi | \phi_1 \wedge \phi_2 | ... | \Box\phi | \Diamond\phi$$

Duality

Conjunctive	Disjunctive	
$\wedge$	$\vee$	$\neg\wedge \leftrightarrow \vee\neg$
$\forall$	$\exists$	$\neg\forall \leftrightarrow \exists\neg$
$\Box$	$\Diamond$	$\neg\Box \leftrightarrow \Diamond\neg$

Semantics: Possible Worlds Model

$M = \langle W, R, V \rangle$ where:

- W is a set of 'worlds', states
- R is a relation between $w \in W$
- Valuation fun $V(p, w)$ assigns truth value to p in $w \in W$

Then:

$$M, s \models \Box\phi \text{ iff for all } t \text{ such that } R(s,t) : M, t \models \phi$$

$$M, s \models \Diamond\phi \text{ iff there is } t \text{ such that } R(s,t) \text{ and } M, t \models \phi$$

Proof Theory

- Propositional modal logic is identical to dyadic FOPL
- Monadic predicates ‘annotate’ worlds with the statements that are valid in them.
- Box and diamond can (unsurprizingly) be reduced to quantifiers over world instances.

Multi-Modal Logic

$M = \langle W, R, V \rangle$ where:

- W is a set of 'worlds', states
- R_i is a set of relations between $w \in W$
- $L(w)$ maps worlds to valuation functions

Then:

$$M, s \models \Box_i \phi \text{ iff for all } t \text{ such that } R_i(s, t) : M, t \models \phi$$

often also written as:

$$M, s \models K_i \phi \text{ iff for all } t \text{ such that } R_i(s, t) : M, t \models \phi$$

and often read as 'i knows that ϕ'

So, what's the point?

- A system that reaches a state with property p, must/might reach a state with property q
- An agent that knows p, must/might also infer q
- It is the theoretical infrastructure underlying reasoning about FSAs, time, verifying programs and circuits, designing game strategies, verifying traffic lights, and more.

Domain Theories

Semantics of more complex domains: other functions over the elements of the domain besides equality, other algebras for the logical connectives

Unique Name Assumption

- Symbols inequality does not imply referent inequality.
- Think of soc sec number and id number as symbols.
- It might get proven that two different symbols refer to the same entity.
- Stress consistent vs. valid

Complex Terms

- Complex terms instead of atomic symbols
- Think of records and fields
- It might get proven that two different records are unifiable

Algebras as semantics of connectives

- Boole
- Probabilistic
- Min/Max (Fuzzy)
- Lukasiewicz

Arithmetic Domains

- Linear Arithmetic
- Theory of Differences
- Theory of Arrays
- Satisfiability Modulo Theory (SMT)

Some More Domain Theories

- Geospatial, spatiotemporal Domain
- Theory of Arrays
- Satisfiability Modulo Theory (SMT)