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Efficient Document Image Segmentation Representation by Approximating Minimum-Link Polygons

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***Abstract*—The result of a document image segmentation task, e.g. text line or word segmentation, is usually a labeled image with each label corresponding to a different segmented region. For many applications, the segmented regions need to be stored and represented in an efficient way, using simple geometric shapes. A challenging task is to restrict all pixels corresponding to a specific label inside a polygon with a minimum number of vertices. Such a polygon promotes the description simplicity and the storage efficiency, while providing a much more user-friendly representation that can be edited easily. The proposed method is a cost-effective approximation of the minimum-edges polygon problem, computing a contour enclosing only pixels of a certain label and using a greedy algorithm in order to reduce the contour into a minimum-link polygon that retains the separability property between the labeled set of pixels.**

***Keywords*—Document Image Segmentation Representation, Groundtruth Representation, Minimum-Link Polygon Approximation**

I. Introduction

A basic preprocessing step in the document analysis and recognition pipeline is image segmentation to specific elements, such as regions, text lines, words or even characters. The majority of the approaches for document image segmentation that have been proposed in the literature, result in a labeled image that represents different segmented regions, according to the segmentation task (e.g. region, text line or word segmentation) [1],[2]. Labeled images, though descriptive and easy to construct as a segmentation task output, are difficult to represent and need many storage resources.

In order to address the problem of defining a simple and efficient representation, it is imperative to restrict all labeled elements inside simple geometric shapes, such as polygons defined by their vertices. This approach provides a simple conceptual representation for each segmented region and results to a very compact and storage efficient representation of the document image segmentation results. Furthermore, this representation is useful for user interaction tasks, such as quickly evaluating or even editing a segmentation result by simply moving some of the boundary polygon vertices. These editing actions are supported by ground-truthing and transcription applications, such as the Aletheia tool [3] and Transkribus [4], and can be further simplified by using less points to represent the segmented regions.

A convenient representation is to use the surrounding rectangular box (bounding box) of every distinct segmented region, which corresponds to storing only four points, the vertices of the bounding box. Another convenient representation is to use the convex hull of every distinct segmented region, which results in storing a larger set of points (the vertices of the convex hull polygon) with the benefit of a "tighter" representation of the segmented region. These approaches are usually inadequate due to possible overlaps between the boxes/convex hulls and consequently this results to poor representation of the elements that cannot be separated anymore. Overlapping convex hulls of segmented regions is a common case in handwritten documents (see Fig. 1) and thus it is important to address the problem of correctly representing segmented regions in an efficient way.

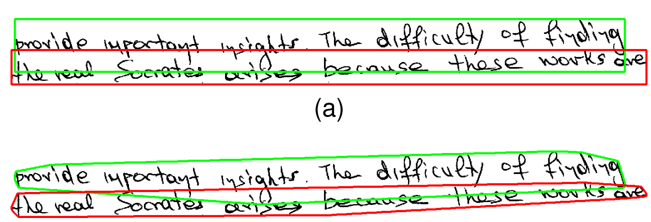

Fig. 1. Two text lines with overlapping (a) bounding boxes (b) convex hulls.

In [5], isothetic polygons are proposed for document segmentation representation. These are polygons that have only

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horizontal and vertical edges or equivalently correspond to the union of a set of rectangles. An isothetic polygon is constructed by iteratively adding and subtracting simple rectangular blocks in order to enclose only the pixels of a specific label, while trying to keep the edges as few as possible. Starting from the bounding box of the selected label, at each step, the minimum-edge rectangles that enclose pixels from another label are subtracted and consequently the minimum-edge rectangles that enclose the pixels which belong to the selected label and are not in the current polygon are added.

However, the use of isothetic polygons approach has some obvious drawbacks, even though it may work sufficiently well for several segmentation scenarios. Specifically, when the segmented regions are very close to each other, isothetic polygons will converge after many iterations and the final representation will consists of a large number of points due to the restriction of horizontal/vertical edges resulting in a procedure similar to contour-following. It can be easily observed that the time and storage complexity of this approach is heavily dependent on the spatial arrangement of neighboring regions.

In this paper, a novel approach for the polygonal representation problem is proposed, trying to address the aforementioned drawbacks of isothetic polygons. The main goal of our approach is to loosen the restriction of vertical and horizontal edges and allow any polygon as a representation, thus giving a more generic and robust description that further decreases the number of storage points. Our approach is based on a two-step scheme. First, we extract a contour that encloses only the pixels of a selected label using Distance Transform [6]. At the next step, a greedy algorithm is used in order to construct the final polygonal representation by retaining as few points as possible from the contour.

The remainder of the paper is organized as follows. In Section II, we discuss about the formalization and the complexity of the problem. In Section III, the proposed method is described in detail, while some applications and examples are presented in Section IV. Finally, conclusions are drawn in Section V.

II. Problem Description

For the rest of the paper we define two different sets of pixels. The first set, denoted as Point Set A or Interior Point Set, consists of the pixels corresponding to a selected label, while the second set, denoted as Point Set B or Exterior Point Set, consists of any other labeled pixel. The background with no label (or labeled as zero) is the "free" space, where a polygon can expand without a problem.

For the trivial case of non-overlapping convex hulls or bounding boxes between every pair of the segmented regions, a satisfying solution would be the convex hull or the bounding box accordingly.

Our goal is to build upon the simplicity of the convex hull representation by finding the smallest (non-convex) polygonal envelop containing only the points that belong to a specific label, as it is stated below:

Minimum Separating Polygon: *Given a Set of Interior Points $A = \{a_1, a_2, ..., a_N\}$ and a Set of Exterior Points $B = \{b_1, b_2, ..., b_K\}$, find a Polygon P with the minimum Perimeter such that every a_i is inside P and every b_i is outside P or, equivalently, find the minimum-perimeter Polygon P that separates the two Sets.*

A special case of the above problem occurs when none of the points in Set B lies inside the convex hull of Set A. In this case, the Minimum Separating Polygon is in fact the Convex Hull of the Set A, which can be computed in $O(nlogn)$ (Graham Scan [7]). However, for the general case, the problem is *NP-hard*, as shown in [8].

Due to the fact that the initial problem belongs to the NP-hard complexity class, we choose to change the minimum perimeter optimization goal and replace it with the optimization goal of minimizing the number of the polygon edges (or equivalently the number of vertices), denoted as minimum-link goal in the literature, which is more efficient in terms of storage requirements. Furthermore, we add a new constraint to the bounding area of the resulting polygon because unconstrained minimum-link optimization may result to flawed representations that are far off the initial goal of minimum-perimeter polygons (see Fig. 2). Therefore we constrain the minimum-link polygon to lie into the area defined by a bounding (convex) polygon C. In practice, an expanded bounding box is used as the bounding polygon C, which promotes a more descriptive representation as it is depicted in Fig. 3. The above problem is formulated as follows:

Minimum-Link Representation Polygon: *Given a Set of Interior Points $A = \{a_1, a_2, ..., a_N\}$, a Set of Exterior Points $B = \{b_1, b_2, ..., b_K\}$ and a Bounding Polygon C, find a Polygon P inside C with the minimum number of edges/links such that every a_i is inside P and every b_i is outside P.*

The above problem is also a separating problem between two sets (the boundary points of the bounding polygon C can be also considered points of the exterior point set) with the goal of optimizing the edges of the separating polygon. However, this problem still belongs to NP-hard complexity class [9].

As it will be described in detail in the upcoming section, we propose an efficient approximation, with linear complexity in terms of the image size, to the later problem in order to represent a segmented region using a polygon to separate the points of the label of interest from any other labeled points. The proposed approach assumes that such a polygon exists.

III. Proposed Methodology

The proposed methodology emphasizes on finding an approximation of the minimum edge (restricted) polygon at a low computational cost, using a two-step scheme: **a)** estimate a contour enclosing only the label of interest **b)** efficiently reduce the contour to a valid polygon with as few edges as possible. Segmented region representation has not a strict requirement over the number of polygon vertices and it will be beneficial if the whole procedure was fast enough as part of an application or an interaction tool. Therefore, we prefer

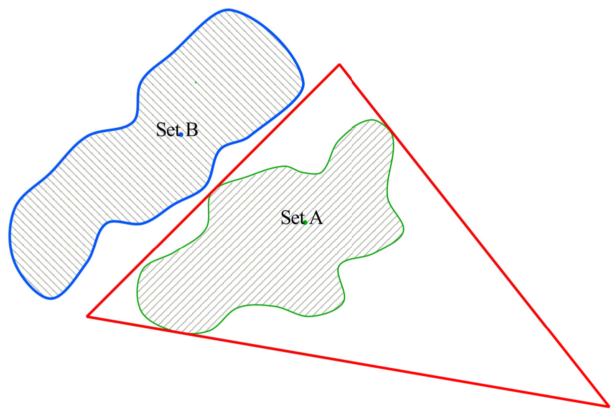


Fig. 2. Example of a non-descriptive enclosing polygon with minimum edges.

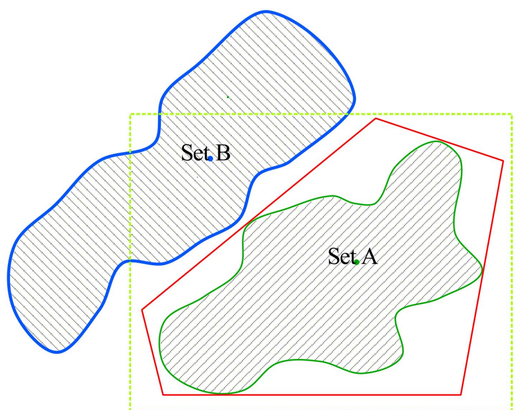


Fig. 3. Example of an enclosing polygon with minimum edges restricted in the (dashed) bounding polygon C.

taking a rough estimation of a contour that ideally can produce a minimum-link polygon over choosing a more "rational" algorithmic approximation which may be significantly slower.

The area of interest for each label is the interior of the expanded bounding box of the corresponding pixel set. As it was mentioned before, this choice is done in order to restrain the resulting polygon into a representative area for the labeled region and avoid flawed non-descriptive representations.

The proposed approximation utilizes the distance transform in order to find a contour that is maximally distanced from the interior and exterior sets of points, i.e. that maximizes the margin of the separating contour. The maximized margin contour, that is a common practice in separating/optimizing techniques, is a choice that promotes the reduction of the contour to a polygonal boundary with few edges. Having maximized this margin, each polygon edge has more "space" to expand into both directions, inwards and outwards, and subsequently the greedy selection of the polygon vertices in the next step is further assisted.

Given the contour that encloses only the pixels of a certain label, the final polygon is produced by a minimum collection of contour points as vertices in order to form a valid polygonal boundary without including any other label or excluding pixels of the selected label. This is achieved by a greedy algorithm that passes through each point of the contour and decides either to discard or keep it according to the validity of the line segment starting from the previous kept point of the contour and ending at the current point. A detailed description of the aforementioned algorithm is provided in the respective subsection.

A. *Contour Extraction*

Given a selected label l, we extract an expanded bounding box of the labeled region, the area of interest, which corresponds to the part of the image that the following procedures will be applied. The bounding box is chosen to be expanded (by a fixed number of pixels at each direction) in order to further assist the maximization of the margin between the computed contour and the point sets. The contour extraction is based on the inner distance transform $IntDist_l$, i.e. the smallest euclidean distance from the interior point set, and the outer distance transform $ExtDist_l$, i.e. the smallest euclidean distance from the exterior point set, including the boundary of bounding box. These distance transforms are computed only at the respective area of interest.

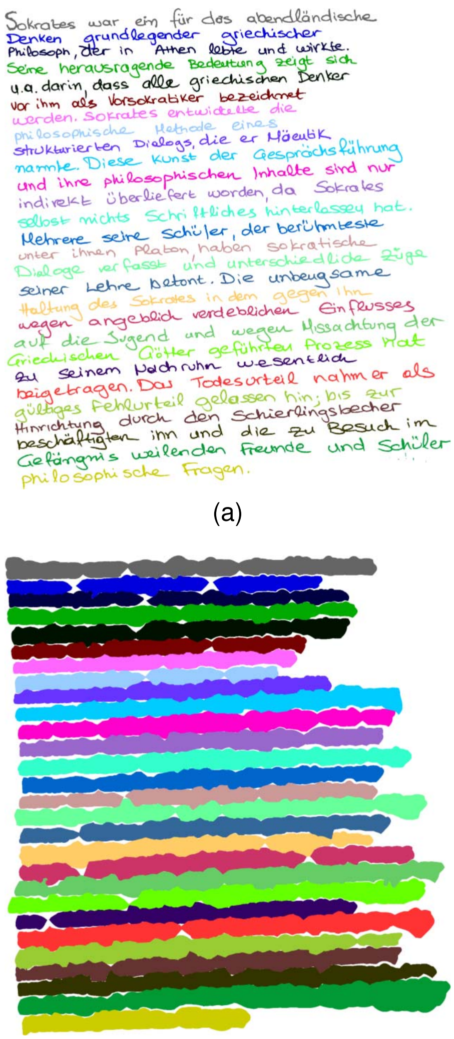
The contour is computed as the boundary of the label's inner shape, which is defined as the area that is closer to the inner distance transform than the outer distance transform. Formally, the inner shape for a label l is defined as the foreground of the binary image $BW_l = IntDist_l < ExtDist_l$, and consequently its boundary is consisted of pixels that satisfy the condition $|IntDist_l - ExtDist_l| < 1$. The later condition implies that the proposed boundary maximizes the distance between of the contour and both the interior and the exterior set of points, as it was mentioned before. The computed inner shape is in fact an non-uniform dilated version of the initial labeled regions, while it preserves the separation (between points of different labels) constraint, as it can be depicted in Fig. 4.

However, in order to perform the greedy algorithm of the second step we need a simple cohesive contour for representing each region and not a set of independent contours. The aforementioned procedure for extracting the boundary of a region may lead to multiple independent contours due to sparse neighborhoods of interior points, e.g. words that are far apart.

A simple solution to the aforementioned problem, which additionally promotes results similar to convex hull, is to apply a two-pass approach. At the first pass we compute the inner shape BW_l (Fig. 4b) and the convex hull CH_l for each label. At the second pass we update the inner shape BW_l' by merging the convex hull CH_l and the initial shape BW_l and subtracting any other label's shape according to following relation:

$$\begin{aligned} BW_l' &= \cap_{i \neq l}[(CH_l \cup BW_l) \setminus BW_i] \\ &= \{x \in CH_l \cup BW_l : x \notin BW_i, i \neq l\} \end{aligned}$$

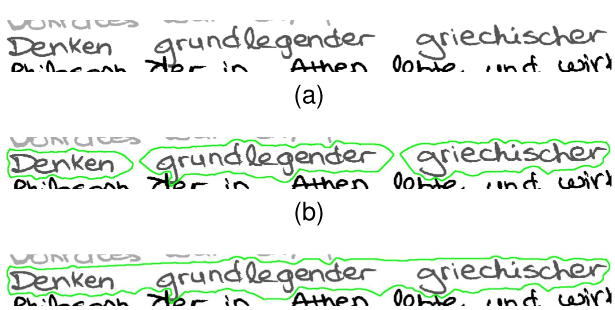
The results of this approach and the corresponding contour are depicted in Fig. 5.



(a)

(b)

Fig. 4. (a) Initial labeled image (output of a text line segmentation task) and (b) the result of the inner shape procedure



(a)

(b)

(c)

Fig. 5. (a) Initial part of labeled image (output of a text line segmentation task), (b) the result of the inner shape procedure and (c) the result of the merging procedure that uses the convex hull

It should be noted that even the use of convex hull does not guarantee the existence of a single boundary and multiple independent inner shapes may exist. In this case, we simply merge the extracted contours by finding the shortest paths between them. However, for well structured regions, like the result of a document image segmentation, this is very rare and can only be found in cases of "bad" segmentation results, where regions are highly entangled with each other and not well-separated.

B. Greedy Selection of Polygon Vertices

The selection of the minimum number of vertices/edges is approximated by a linear greedy algorithm. Starting from the left-most point (it can be chosen at random) of the contour we follow it clockwise and we discard as many points as it is possible. The discarding procedure is performed by checking if a line segment defined by two points of the contour is a valid separating boundary, i.e. does not exclude interior points or include exterior. Assuming that the start of the line segment is a member of the final polygon we discard successive points of the contour that form a valid line segment, until we found one that violates our condition. This point will be added to the final polygon and will be the new starting point of the next line segment to be checked. It is clear that the proposed approach tries to replace parts of continuous points of the contour with line segments in order to reduce the initial contour to a polygon with few edges.

The validity check of a line segment is not a simple task, as we should compare the line segment with every point of the interior and exterior set. In order to address efficiently this problem we utilize the previously computed inner and outer distance transforms $IntDist$, $ExtDist$. The value of the distance transform at a point of the contour defines a circular area around that point, where a line segment can lie inside without violating the validity. The radius of the circular area is the value of the distance transform at each point. For simplicity, we assume that each point (x, y) has only one distance transform value, $\min(IntDist(x, y), ExtDist(x, y))$. The overall area that a line segment is permitted to lie within is denoted by the union of the circular areas corresponding to the consecutive points of the contour, from the starting point until the current checked one.

The above formulation of the permitted space can be utilized in a fast and easy way in order to validate line segments by using a simple $O(1)$ check of the maximum permitted rotation θ of the line segment. Such an angle is computed as an overestimation of its real absolute value by supposing that the point (interior or exterior) that is nearest to the current point of the contour is perpendicular to the current line segment, as it is depicted in Fig. 6. The overestimation guarantees the credibility of the resulting polygon to be a separating one. Therefore, at each point we compute the temporary maximum angle $t\theta$ that the current line segment can be rotated. If the current angle $t\theta$ is greater than the maximum permitted one (θ) the line segment is not valid, otherwise we update the maximum permitted angle ($\theta \leftarrow t\theta$).

Finally, we increase the flexibility of the resulting polygon by having two distinguished validity checks at each iteration,

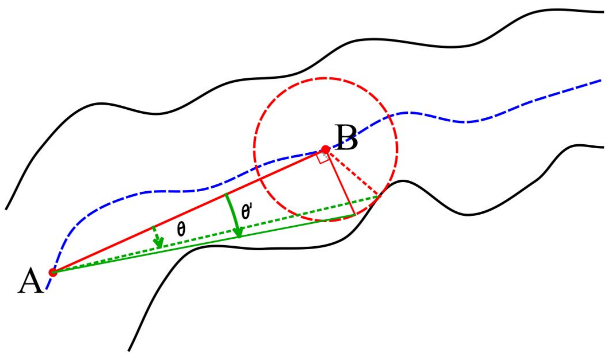


Fig. 6. Validity check of a line segment by overestimating the margin angle

one for each direction, by utilizing both (inner and outer) distance transforms. Summarizing, given a start point, at each step we compute an overestimation of the angles that the expanding line segment can be rotated for both directions, inwards and outwards, and we discard consecutive points until the condition for the maximum permitted angle is not met. The proposed algorithm is summarized in Fig. 7.

C. Complexity of the Algorithm

The computational complexity of the proposed methodology is the summation of the complexity of the two steps. The first step, that is dominated by the distance transform computation, has a $O(n \cdot m)$ complexity for a $n \times m$ bounding box [6]. In the case of the merged boundary with the convex hull the complexity is $O(n \cdot m + N_A log N_A)$, where N_A is the number of interior points. The second step is in fact a greedy algorithm that traverses the contour produced by the previous step and uses the already computed distance transform in order to efficiently compute the final polygon. The greedy tour over the contour visits each point at most two times so the algorithm is linear to the input size N_C, i.e. the number of contour points ($N_C = O(n \cdot m)$). Therefore a representation polygon is computed in $O(n \cdot m + N_A log N_A + N_C)$, which is comparable to the complexity of simply reading the initial image and computing the convex hull (proportional to a constant factor).

The memory requirements are $O(N \times M)$, i.e. the size of the image, and correspond to storing the distance transform maps from step one in order to utilize them in step two and avoid re-computing them.

IV. Applications and Examples

One of the most common ways of storing groundtruth and segmentation results is by using a pixel-based representation, in the form of labeled images. However, labeled images are not such a practical representation for tools/applications that require the user to interact or supervise the results.

The polygon representation is a much more user-friendly representation due to the fact that the area of interest can be highlighted in a very simple manner. Additionally, a polygon representation with few edges can be modified easily in order to correct the area of interest if needed. Thus, polygon

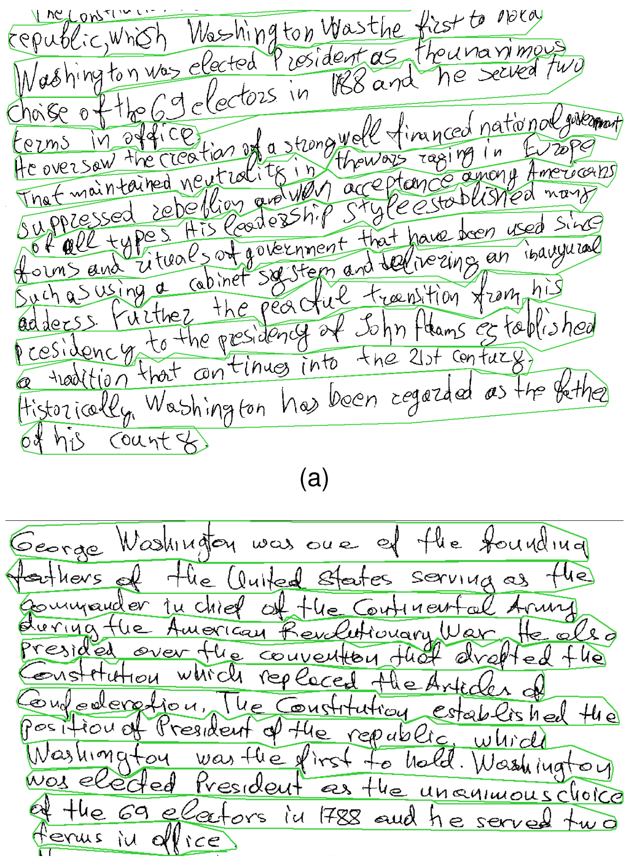
```
procedure MinPolygon
    input: Distance Transforms IntDist, ExtDist and
           Contour Points C
    output: Polygon Points P
    i ← 0, cnt ← 0, N ← length(C)
    while i < N do
        a_s ← C[i]
        j ← i + 1
        id_in ← j,    id_out ← j
        θ_in ← −2π,    θ_out ← 2π
        while j < N do  ▷ find next invalid line segment
            a_t ← C[j]
            θ ← ∠(a_t, a_s);
            if θ ≤ θ_in then
                P[cnt] ← C[id_in]
                cntv ← cnt + 1, i ← id_in
                break
            end if
            if θ ≥ θ_out then
                P[cnt] ← C[id_out]
                cnt ← cnt + 1. i ← id_out
                break
            end if
            tθ_in ← θ − tan^{−1}(IntDist(a_t)/d(a_s, a_t))
            tθ_out ← θ + tan^{−1}(ExtDist(a_t)/d(a_s, a_t))
            if tθ_in > θ_in then
                θ_in ← tθ_in, id_in ← j
            end if
            if tθ_out < θ_out then
                θ_out ← tθ_out, id_out ← j
            end if
            j ← j + 1;
        end while
    end while
    return P
end procedure
```

Fig. 7. Greedy Algorithm for Approximating Min-Link Polygon

representation assists applications that rely on the user interference and has practical use in representation/groundtruthing tools such as Aletheia [3], where each segmented region is represented as a polygon and is stored as a set of points (polygon vertices) in an xml file.

Therefore, an obvious advantage of our approach is the minimization of the needed storage, as it can produce smaller xml files. Besides the benefits in terms of storage, our approach is a very fast approximation of the minimum-link problem and its complexity is independent of the elements structure in contrast to isothetic polygons approach [5], which converges slower for more complex neighboring elements.

Examples of word and text line representation are shown in Fig. 8, 9. The initial images and the labeled images (groundtruth represented as .dat files) were part of the ICDAR 2013 Handwriting Segmentation Contest [1]. The results are presented using the Aletheia tool, which overlays the polygons, stored at an xml file, over the initial image.



(a)

(b)

Fig. 8. Examples of polygonal representation for a text line segmentation task

V. Conclusion

A novel method for document image segmentation representation is proposed. Segmented regions are represented by approximating a minimum-link polygon that contains only pixels of the respective region, while trying to restrict the final polygon into a "tight" area around this region for a more descriptive representation. The proposed approach is efficient in terms of computational cost, by taking advantage of certain geometrical properties of the problem, as well as in terms of storage, as each segmented region is represented by the vertices of the resulting polygon.

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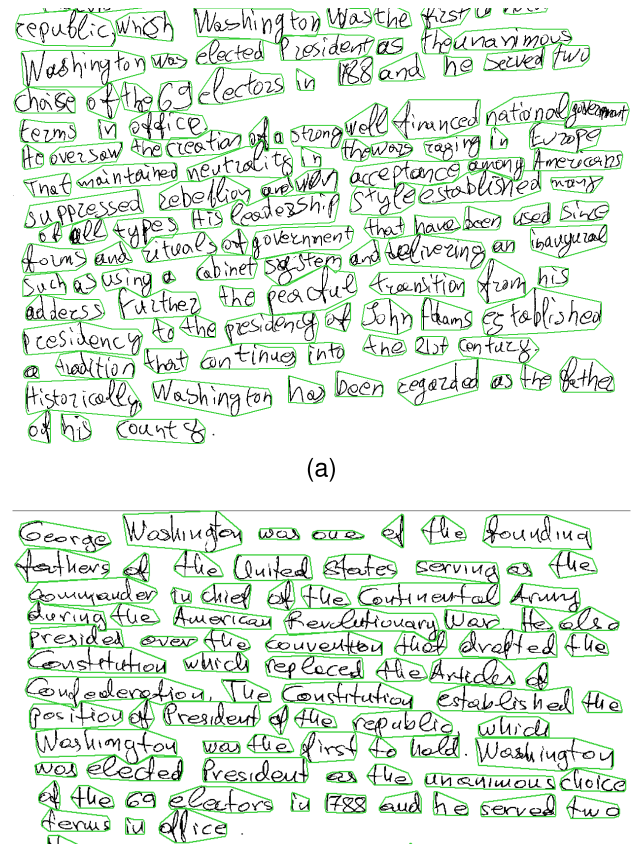

(a)

(b)

Fig. 9. Examples of the proposed polygonal representation for a word segmentation task

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