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STATIC POTENTIALS FROM PURE SU(2) LATTICE GAUGE THEORY

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The potentials between static fundamental colour sources corresponding to the ground state and a number of excited states of the gluon field are studied through a Monte Carlo simulation of SU(2) lattice gauge theory with a technique involving "blocked" paths of spatial links The results are for β-values of 2 4 and 2 5 and for source separations of up to 1 1 fm We obtain values for the potentials corresponding to the lowest mode for seven irreducible representations of the symmetry group of the gluon field and also to the first excited mode for two of the representations The lowest excited potential corresponds to the E_u symmetry of the gluon field The ground state potential exhibits non-perturbative scaling at the β-values studied, but the value of the ratio of lattice spacings is in disagreement with the perturbation theory prediction The excited potentials scale according to the ground state scaling within the errors String model predictions for the potentials are compared with our results, similarities and discrepancies are discussed

1 Introduction

The potential between heavy fundamental colour sources is an important non-perturbative feature of QCD The evaluation of this potential in the ground and possible excited states of the gluon field is of relevance to hadron phenomenology In particular, the potentials corresponding to the lowest gluon field excitations are believed to determine the spectrum of the low-lying heavy quark hybrid mesons, some of which are predicted to have exotic quantum numbers

The ground state potential has been accurately determined from lattice gauge theory studies, at least in the theory without dynamical fermions Excited potentials on the lattice have been studied in the past [1–3] through various techniques and qualitative conclusions have been drawn However, the relatively large statistical and systematic errors in these calculations do not allow for an accurate description of the potentials For example, it is not clear which is the lowest lying of the excited potentials

Recently, the technique of constructing "fuzzy" or "blocked" operators [4] has proved useful for glueball calculations and for string tension measurements from Polyakov loop correlations [4, 5] In this paper we combine this method with a

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variational technique and the use of multihit links [6] in the time direction in order to calculate the potentials of the ground state and of a number of excited states for the heavy source–antisource system in the context of SU(2) gauge theory without dynamical fermions Although a similar study of SU(3) is feasible, results from the two gauge theories for static fundamental source potentials are believed to be similar and SU(2) is easier to treat computationally

We determine the potentials by the usual method of evaluating correlations between linear combinations of paths which are constructed from spatial links and displaced in the temporal lattice direction The "blocking" technique enables us to extract reliable results from small temporal separations, where the Monte Carlo signal-to-noise ratio is big and the statistical errors are small We evaluate the potentials corresponding to the lowest lying eigenmodes in most of the irreducible representations of the symmetry group D_{4h} of the gluon field between the sources The small statistical errors of our results compared with previous work [3] allow us to demonstrate that the lowest lying excitation corresponds to the E_u representation of D_{4h} for spatial separations of the sources between 0 1 and 1 1 fm Non-perturbative scaling is confirmed for $\beta = 2\ 4$ and $\beta = 2\ 5$ for the ground state potential and a number of excited potentials

It is expected that the heavy quark potentials should be adequately described by a relativistic string model for relatively large separations of the sources It is thus interesting to see how our results compare with string model predictions In particular, for the excited potentials we shall consider a Nambu string with fixed ends representing the positions of the static sources and compare its spectrum with our results

This paper is organized as follows In sect 2 we describe the method and give details about the statistics of our Monte Carlo calculation In sect 3 we present our results, which we compare to string model results in sect 4 We draw our conclusions in sect 5

2. Monte Carlo simulations

We use the Wilson action on L^4 lattices with periodic boundary conditions The potential corresponding to the ground state of a given irreducible representation $\mathscr{R}$ of D_{4h} is given by the relation

$$V_0 = -a^{-1} \ln \lambda_0 = -a^{-1} \lim_{t \to \infty} \ln\left[W_{ij}(t+1)/W_{ij}(t) \right], \qquad (2\ 1)$$

where $W_{ij}(t)$ denotes the correlation between operators O_i and O_j transforming as the representation $\mathscr{R}$ and separated by t lattice units in the temporal direction, and λ_0 is the largest eigenvalue of the transfer matrix $\hat{T}$ in this representation To calculate these correlations we use multihit time-like links [6], i e we replace the

TABLE 1
The irreducible representations of D_{4h}, the corresponding continuum quantum numbers K^{PC} for the minimum value of the angular momentum K along the quark–antiquark axis and the J^{PC} quantum numbers of the quark–antiquark state with minimum angular momentum

		J^{PC}			
D_{4h} rep	K^{PC}	$S=0$	$S=1$		
A_{1g}	0^{++}	0^{-+}	1^{--}		
A_{2g}	0^{--}	$\underline{0}^{+-}$	1^{++}		
A_{1u}	0^{-+}	0^{++}	1^{+-}		
A_{2u}	0^{+-}	$\underline{0}^{--}$	$\underline{1}^{-+}$		
B_{1g}	2^{++}	2^{-+}	1^{--}	2^{--}	3^{--}
B_{2g}	2^{--}	$\underline{2}^{+-}$	1^{++}	2^{++}	3^{++}
B_{1u}	2^{-+}	2^{++}	1^{+-}	$\underline{2}^{+-}$	3^{+-}
B_{2u}	2^{+-}	2^{--}	$\underline{1}^{-+}$	2^{-+}	$\underline{3}^{-+}$
E_u	1^{+-}	1^{++}	$\underline{0}^{+-}$	1^{+-}	$\underline{2}^{+-}$
	1^{-+}	1^{--}	0^{-+}	$\underline{1}^{-+}$	2^{-+}
E_g	1^{++}	1^{+-}	0^{++}	1^{++}	2^{++}
	1^{--}	$\underline{1}^{-+}$	$\underline{0}^{--}$	1^{--}	2^{--}

Underlined values represent non-quark model quantum numbers

links in the temporal direction by their thermal average in the fixed environment of the six surrounding U-bends The irreducible representations of D_{4h} are listed in table 1 The notation is that A, E and B correspond to minimum angular momentum about the source–antisource axis of 0, 1 and 2 respectively in the continuum limit, 1 (2) refers to symmetry (antisymmetry) under interchange of ends by a rotation by π about a lattice axis and g (u) refers to symmetry (antisymmetry) under interchange of ends by inversion in the midpoint

For finite t the ratios $\lambda(t+1,t) = W_{ij}(t+1)/W_{ij}(t)$ receive contributions from excited states However, if the operators O_i and O_j have a high enough overlap with the ground state, λ_0 and V_0 can be reliably determined from relatively low values of t, where the Monte Carlo noise is small To create such operators from our initial paths, we use a "blocking" algorithm We construct linear combinations of the paths shown in fig 1, suitably chosen to transform like each of the irreducible representations of D_{4h} From each of the spatial paths we form new paths by replacing each spatial link by a multiple of itself plus a sum of the four neighbouring spatial U-bends

$$U_\mu(n) \to U_\mu^1(n) = A_n\left\{ cU_\mu(n) + \sum_{\substack{\pm\nu\neq\mu \\ \nu\neq 4}} U_\nu(n)\, U_\mu(n+\hat{\nu})\, U_\nu^\dagger(n+\hat{\mu}) \right\} \tag{2 2}$$

Here c is a free parameter and A_n is a normalisation factor chosen to project U_μ^1

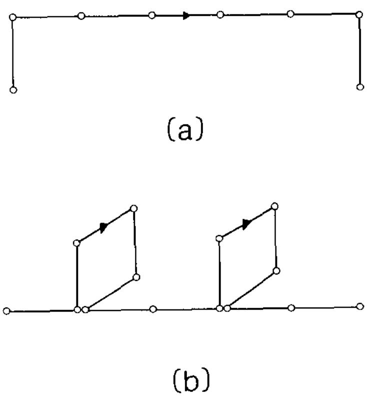

Fig 1 Paths used to form linear combinations transforming like the irreducible representations of D_{4h} (a) U-bend paths The first link of the path can be in the direction of any spatial lattice vector perpendicular to the source–antisource axis (b) These paths can have any number of square "staples" with their first link parallel to any of the spatial lattice vectors perpendicular to the source–antisource axis and the electric flux running clockwise or anticlockwise

into SU(2) This "blocking" procedure can be iterated to higher levels At each level the newly formed paths transform according to the same D_{4h} representation as the original paths The blocking process, iterated to a sufficiently high level, is an efficient method for creating linear combinations of paths with spatial extension which are likely to have a high overlap with the lowest lying state within a given D_{4h} representation The constant c in eq (2 2) and the total number M of iterations of the blocking algorithm (blocking level) are free parameters which can be adjusted to optimize the path overlap with the lowest state in a given D_{4h} representation For each representation, the ratio $r_{10,21} = \lambda(1,0)/\lambda(2,1)$ is a measure of the overlap with the lowest state (the larger $r_{10,21}$ the higher the overlap, if an operator overlaps only with the lowest state, the corresponding $r_{10,21}$ is equal to 1) Hence $r_{10,21}$ can be used to determine the optimal values for c and M The optimal values depend on the value of β, on the lattice separation R/a of the sources and on the D_{4h} representation Since we want to allow for simultaneous measurement for many group representations at a given β value, we cannot use the exact optimal values of c and M for each representation and R/a separately A preliminary low statistics study of the representations A_{1g}, E_u and A_{1u} at $\beta = 2.4$ and 2.5 showed that with $c = 2.5$ we can find two values M_1 and M_2 of M corresponding to values of $r_{10,21}$ very close to the optimal value for all three representations and all studied values of R/a We thus create two independent paths P_{M_1} and P_{M_2} transforming according to the same group representation and we measure the four correlations

$$C_{ij}(t) = \langle P_{M_i} | \hat{T}^t | P_{M_j} \rangle, \quad i, j = 1,2, \quad t = 0,1,2, \tag{2.3}$$

between these paths separated by t steps in the temporal direction We analyse these correlations by a matrix variational technique to obtain, for each symmetry of the gluonic configuration, estimates for the two highest eigenvalues λ_0 and λ_1 of the transfer matrix and for the corresponding values of the potentials The variational technique amounts to finding a path combination which maximizes the overlap with the ground state To avoid the possibility of statistical errors influencing this selection of an optimal linear combination of the paths, we use the optimal combination obtained for comparing path correlations with temporal separations 0 and 1, where the statistical errors are smallest We then obtain our estimates for the potentials from larger t-separations using this fixed linear combination

Having obtained variational estimates $\lambda_{\text{var}}(t+1, t)$ for the eigenvalues of the transfer matrix from analysing ratios of the form $C_{ij}(t+1)/C_{ij}(t)$ at successive values of t ($t = 0, 1, 2,$), we then choose as our final estimate the value corresponding to the lowest value t_0 for which the difference $\Delta = |\lambda_{\text{var}}(t_0+1, t_0) - \lambda_{\text{var}}(t_0, t_0-1)|$ is less than the statistical error for $\lambda_{\text{var}}(t_0+1, t_0)$ If the range of values of t studied is not broad enough for this to happen, Δ represents an upper systematic error in our estimate Typical values of t_0 are $t_0 = 2$–4 Other methods of extracting a final estimate for the eigenvalues (e g two exponential fits) have lead to very similar results for the eigenvalues and the errors attributed to them

We present results at two β values ($\beta = 2\ 4$ and $\beta = 2\ 5$) The lattice sizes and details of the statistics and measured Wilson loops are presented in table 2 For $\beta = 2\ 4$ we have three sets of results each for different linear combinations of the paths of fig 1 The third set of lattices is the only one for which we have attempted measuring the potentials for all D_{4h} representations For the first, second and fourth set we only used paths overlapping with representations A_{1g}, A_{1u} and E_u We divide the results from each set of lattices into blocks, average the results of each block and do error analysis on the set of averages The blocks are chosen large enough, so that the block averages show no autocorrelation We find that the results from the three sets of lattices at $\beta = 2\ 4$ are compatible In sect 3 we present the results with the smallest statistical errors

TABLE 2

Details of our Monte Carlo simulations and measurements for the potentials

β	Size of lattice	Number of updates	Lattices measured	ι	Blocking levels	Range of R/a	Range of t
2 4	16^4	24000	1200	2 5	12	$2 \le R/a \le 7$	$0 \le t \le 4$
2 4	16^4	10000	1000	2 5	12, 16	$2 \le R/a \le 8$	$0 \le t \le 5$
2 4	16^4	12000	400	2 5	10, 14	$2 \le R/a \le 8$	$0 \le t \le 5$
2 5	20^4	12000	600	2 5	29, 34	$R/a = 2m$, $1 \le m \le 6$	$0 \le t \le 5$

3. Results

We first present the results for the ground state of the gluon field between static sources This is the lowest state with A_{1g} symmetry At a single β value, we can fit the results for the potential $aV(R/a)$ using the relation

$$aV(R/a) = kR/a - eG(R/a) + C \tag{3 1}$$

with $k = Ka^2$ where K represents the string tension Here we have substituted the usual Coulomb term $-ea/R$ by a multiple of the scalar lattice propagator $G(R/a)$ on a $(2L)^4$ lattice This substitution has been suggested in ref [3] as a natural way of taking into account lattice artefacts which lead to asymmetries between on-axis and off-axis potentials We find excellent fits to eq (3 1) for both β values studied To test for scaling we fit at the two β values simultaneously using the same value of e and assuming that the coefficients of the linear term obey the scaling relation

$$k_{2\,4}/a_{2\,4}^2 = k_{2\,5}/a_{2\,5}^2 = K \tag{3 2}$$

We find an excellent combined fit ($\chi^2 = 0\ 3$) with the values

$$Ka_{2\,4}^2 = 0\ 0728(6), \qquad a_{2\,4}/a_{2\,5} = 1\ 435(7), \qquad e = 0\ 240(4),$$
$$C_{2\,4} = 0\ 524(3), \qquad C_{2\,5} = 0\ 526(3) \tag{3 3}$$

The good quality of the fit confirms non-perturbative scaling for the values $\beta = 2\ 4$ and $\beta = 2\ 5$ This is in agreement with the results of Sommer [7] but contrasts to what has been found for the scaling behaviour of perturbatively improved Creutz ratios by Gutbrod [8] However, the value for the ratio of the lattice spacings is not consistent with asymptotic scaling From the ratio of the lattice spacings at two neighbouring values β_1 and β_2 of β an estimate for the beta function can be obtained

$$\beta_g = \frac{g^3}{8} \frac{\beta_2 - \beta_1}{\ln[a(\beta_2)/a(\beta_1)]} \tag{3 4}$$

This formula gives $8g^{-3}\beta_g = -0\ 277(4)$ which does not agree with the value $-0\ 403$ obtained from the two loop perturbation theory expression

$$8g^{-3}\beta_g = -\frac{11}{3\pi^2}\left(1 + \frac{17g^2}{44\pi^2}\right) \tag{3 5}$$

The departure from asymptotic scaling has been observed in Monte Carlo renormalisation group studies [9] and Monte Carlo spectrum and potential evaluations

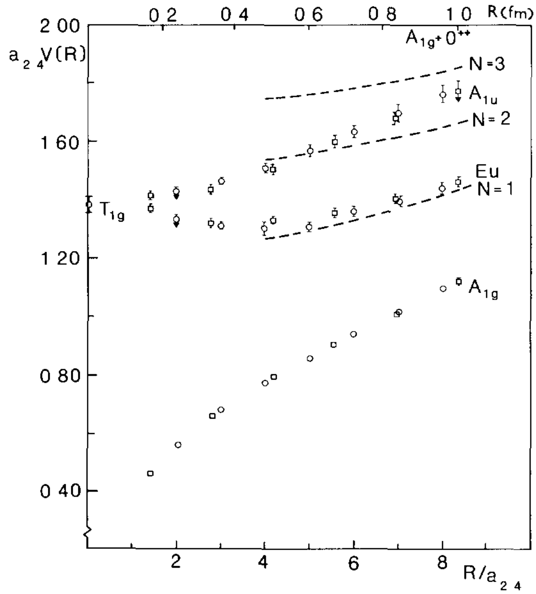

Fig 2 The lowest A_{1g} (ground state), E_u and A_{1u} representation potentials for a static source–anti-source pair of separation R Results for $\beta = 2\,4$ (○) and $\beta = 2\,5$ (□) are plotted using the lattice spacing ratio and lattice self-energy values from a combined fit to the A_{1g} potential Also shown are the energy of the 0^{++} glueball excitation of the ground state potential and the mass of the lowest T_{1g} gluelump (corrected for the difference of the self-energies as discussed in the text) The dashed curves represent Nambu string model potentials

[3, 7, 8, 10, 11] The departure found here agrees with the results in ref [7] for $\beta = 2\,4$ and 2 5 and is somewhat more pronounced than that found in refs [3, 9], where, however, the resolution is coarser

Our results for the potentials corresponding to the ground state and a number of excited states of the gluon field are shown in figs 2, 3 and 4 (the physical distance scale in these figures is set by the value $K^{1/2} = 440$ MeV determined from the ρ, f, Regge trajectory) More specifically, we present results for the lowest lying states with symmetries A_{1g}, A_{1u}, A_{2g}, B_{1g}, B_{2u}, E_u and E_g and for the first excited states with symmetries A_{1g} and E_u We note that the effect of using blocked paths is to increase substantially the overlap of the paths with the ground state of each representation Indeed, with unblocked paths $r_{10\,21}$ is typically 0 50–0 60 for A_{1g} and less than 0 1 for the rest of the representations at $\beta = 2\,4$ In contrast, typical values of this ratio with blocked paths are 0 99 for A_{1g}, 0 81–0 84 for A_{1u}, 0 80–0 83 for A_{2g}, 0 78–0 81 for B_{1g}, 0 71–0 74 for B_{2u}, 0 86–0 90 for E_u and 0 77–0 82 for E_g For the rest of the representations this ratio is typically less than 0 50 and the convergence of the estimate from successive t-values for the highest eigenvalue of the transfer matrix corresponding to these symmetries is relatively

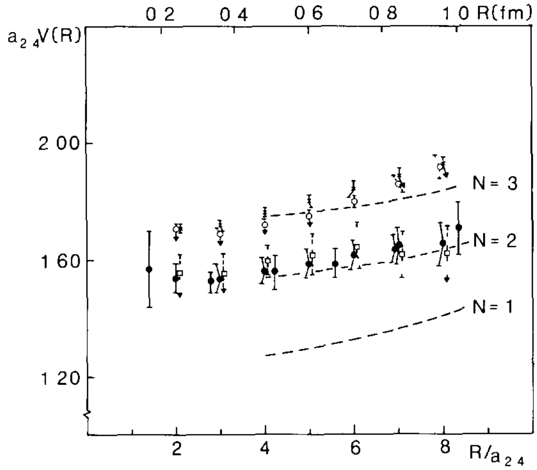

Fig 3 The potentials corresponding to the lowest mode of the gluon field in the B_{1g} (□), E_g (×) and A_{2g} (○) representations and to the first excited mode in the A_{1g} representation (●) The B_{1g}, E_g and A_{2g} results are for $\beta = 2\,4$ while the A_{1g} results are for $\beta = 2\,4$ and $2\,5$ All potentials shown here correspond to the $N = 2$ Nambu string model potential

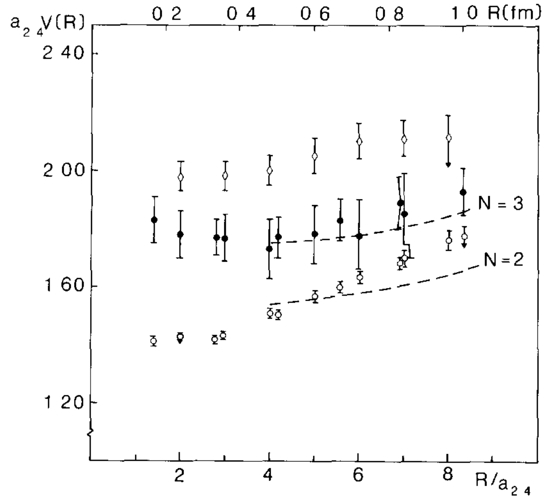

Fig 4 The potentials corresponding to the lowest mode of the gluon field in the A_{1u} (○) and B_{2u} (◇) representations and to the first excited mode in the E_u representation (●) Results for the B_{2u} potential are for $\beta = 2\,4$ and for the A_{1u} and E_u potentials for $\beta = 2\,4$ and $2\,5$ The potentials plotted here correspond to the $N = 3$ Nambu string model potential

poor, so that large systematic errors must be attributed to any estimate of the corresponding potentials

The gluonic configuration transforming as the E_u representation is seen from the data to be the lowest of the gluonic excitations for all the values of the source separation studied It was noted in earlier work of the Liverpool group [3], where the statistical errors were substantially larger, that this state appeared to be approximately degenerate with the state of symmetry A_{1u} In our present work, the potentials are statistically very well determined, as a result of employing the blocking technique and using higher statistics and this apparent degeneracy is resolved We find that the potentials corresponding to the two symmetries are close at low R/a and tend to diverge for larger R/a Given the improved status of glueball mass calculations [5] we can compare the E_u potential with the 0^{++} glueball excitation of the ground state potential Fig 2 shows that the values of the latter lie well above the values of the former We conclude that the E_u potential lies lower than both the potentials of all D_{4h} representations for which we have results and than the masses of the continuum of states lying above the ground state potential plus one 0^{++} glueball mass This conclusion confirms the relevance of the E_u potential to the low-lying meson hybrid spectrum

Using the value for the ratio of the lattice spacings at the two β-values $\beta = 2\,4$ and $\beta = 2\,5$ we can check for scaling in the values of the potentials We have results for both β-values for the lowest lying states with symmetry E_u and A_{1u} and for the first excited states with symmetry A_{1g} and E_u The potentials corresponding to these states are seen to scale according to the ground state scaling within the errors This gives confidence that we are seeing continuum physics

4. Comparison with string model predictions

In a bosonic string model picture of the gluon field configuration between a static quark and antiquark, the ground state potential consists of a term linear in the separation R of the sources and of corrections to this term originating from the zero point motion of the quantised string The leading correction is proportional to $1/R$ and the constant of proportionality has the universal value $-\pi/12$ [12] for a general class of bosonic string theories A string theory is expected to be applicable to the static quark–antiquark system for relatively large R, where the width of the flux tube is smaller than its length On the other hand, at small R an effective Coulomb term dominated by one-gluon exchange also contributes to the potential Note that in our combined fit for the ground state potential the coefficient of the lattice $1/R$ term $-e = -0\,240(7)$ is not compatible with the universal value $-\pi/12 \approx -0\,262$, although the two values are close If we try to force a fit with $e = \pi/12$ upon our data, we obtain an unacceptable $\chi^2 = 26$ If, however, we exclude enough points corresponding to the smaller R values, we can obtain a fit to eq (3 1) with $e = \pi/12$ of similar quality to the fit (3 3) It turns out that this can be achieved by excluding

TABLE 3
Classification of the vibrational excitations of a bosonic string with $N \leq 3$ and the correspondence with D_{4h} irreducible representations

N	$\{n_{m+}, n_{m-}\}$	$\lvert\Lambda\rvert^{\zeta_P \zeta_C}$	D_{4h} reps
1	$n_{1\pm} = 1$	$1^{-+}, 1^{+-}$	E_u
2	$n_{2\pm} = 1$	$1^{++}, 1^{--}$	E_g
	$n_{1\pm} = 2$	$2^{++}, 2^{--}$	B_{1g} B_{2g}
	$n_{1+} = 1,\ n_{1-} = 1$	$0^{++}, 0^{--}$	A'_{1g} A_{2g}
3	$n_{3\pm} = 1$	$1^{-+}, 1^{+-}$	E'_u
	$n_{1\pm} = 3$	$1^{-+}, 1^{+-}$	E'_u
	$n_{1\pm} = 2,\ n_{1\mp} = 1$	$3^{-+}, 3^{+-}$	E'_u
	$n_{1+} = 1,\ n_{2\pm} = 1$	$2^{-+}, 2^{+-}$	B_{1u} B_{2u}
	$n_{1+} = 1,\ n_{2\mp} = 1$	$0^{-+}, 0^{+}$	A_{1u} A_{2u}

The primes indicate excited modes (of a particular symmetry)

just two points, the ones corresponding to $R/a_{25} = 2$ and $R/a_{24} = 2$ We thus find that the formula (3 1) with $e = \pi/12$ works well for relatively small separations of the sources ($RK^{1/2} \geq 0$ 75) A similar result has been obtained by Sommer [7]

We now turn to the excited potentials and investigate to what extent they can be adequately described by a relativistic string model The success of the Nambu string in describing the lowest excited static potentials has been noted in the context of a strong coupling SU(3) calculation in two dimensions [14] Here we consider a Nambu string with fixed ends in four dimensions The corresponding potentials for the different string states are given by the formula

$$V_N = \left(K^2R^2 - \pi K/6 + 2\pi NK\right)^{1/2}, \qquad N = 0, 1, 2, 3, \tag{4 1}$$

Here $N = \Sigma_m m[n_{m+} + n_{m-}]$ where n_{m+} (n_{m-}) is the number of clockwise (anti-clockwise) phonons in the mth mode The string states are classified [15] according to the quantum numbers $|\Lambda|^{\zeta_P \zeta_C}$ where $\Lambda = \Sigma_m(n_{m+} - n_{m-})$ is the angular momentum about the string axis and $\zeta_P \zeta_C = \Pi_m[(-1)^m]^{n_{m+} + n_{m-}}$ Comparison with the continuum quantum numbers K^{PC} of the gluonic field configuration between fundamental sources on a hypercubic lattice (table 1) gives the correspondence between the string states and the D_{4h} irreducible representations as illustrated in table 3 In particular, the first string excitation ($N = 1$) corresponds to the lowest lying mode in the E_u representation of D_{4h}

In figs 2, 3 and 4 the potentials for the relativistic Nambu string corresponding to $N = 1$, 2 and 3 are shown for $R/a_{24} \geq 4$ The value of K (eq (4 1)) is taken from the fit to the ground state potential We note that the differences $V_N - V_0$ are essentially independent of whether we include the string fluctuation term in eq (4 1) and not very sensitive to small variations in the value of K The E_u potential determined by our Monte Carlo results closely follows the $N = 1$ string potential for $R/a_{2\,4} \geq 4$, corresponding to physical distances greater than 0.5 fm. There is also a very good correspondence between the lowest B_{1g} and first excited A_{1g} lattice potentials and the $N = 2$ Nambu string potential for a range of values of R. However, the lowest E_g and A_{2g} lattice potentials do not show signs of converging to the $N = 2$ string potential value and at the largest value of R studied they agree better with the $N = 3$ than the $N = 2$ string model potential. Finally, the values of the lattice gauge theory potentials corresponding to $N = 3$ in the string model (fig. 4) are reasonably high at large R and it is conceivable that they converge to the string model values at higher values of R than those we have studied. In particular, the A_{1u} potential approaches the $N = 3$ string model potential at large R but falls away at smaller R values approaching the lowest E_u potential. This behaviour conforms with the prediction [16] that in the limit $R \to 0$ these two gluonic excitations are degenerate with the same energy as the lowest state of the gluelump spectrum, which transforms like the T_{1g} representation of the group O_h. The lattice energy of the T_{1g} gluelump has been determined as $E_{T_{1g}} a_{2\,4} = 1.56(3)$. This figure contains the unphysical lattice self-energy term of the adjoint source. On subtracting that and adding the lattice self-energy of the fundamental sources included in the values of our potentials (determined by the R-independent term of the fit to the ground state potential) we obtain the value $E_{T_{1g}} a_{2\,4} = 1.38(3)$ which is in good agreement with the projection of the lowest E_u and A_{1u} potentials to $R = 0$, as illustrated in fig. 2.

In summary, the Nambu string model leads to a reasonable description of a number of excited static quark potentials for the quark separation $R \cong 1$ fm, although it appears to fail for some of the potentials, notably the ones corresponding to representations E_g and A_{2g}. In particular, the model offers an excellent description of the E_u potential (which is the most interesting from the phenomenological point of view) for a quark separation greater than 0.5 fm.

We close this section with a word about the non-relativistic string model [15], where the excited potentials are equally spaced above the ground state potential with excitation energies $N\pi/R$, $N = 1, 2, \ldots$. We find that only for the largest values of R we have studied (approximately equal to 1 fm) is the first string excitation consistent with the E_u potential. At these values of R the higher excited potentials for the non-relativistic string model generally lie much higher than their lattice gauge theory counterparts.

5. Conclusions and discussion

In this paper we have studied the ground state and excited potentials between a static quark–antiquark pair in the context of SU(2) lattice gauge theory without dynamical fermions. By using "blocked" links in the spatial directions we have been able to construct operators with greatly improved overlaps with the ground state of each D_{4h} representation and hence to determine the potentials corresponding to most of these representations The E_u potential, which has been determined much more accurately than in previously published work, is the lowest lying excited potential The corresponding excitation energy is much less than the lightest glueball mass Hence the E_u potential is the most relevant to hybrid meson phenomenology We have observed non-perturbative scaling in the region $\beta = 2\ 4$–$2\ 5$ for the ground state potential and for a number of excited potentials within the errors Comparison of our results to the predictions of a relativistic string model has shown that the latter provides an accurate description of the E_u potential for quark–antiquark separations greater than 0 5 fm and a qualitatively reasonable description of a number of higher lying potentials for separations of the order of 1 fm

In the adiabatic heavy quark approximation, the spectrum of the lowest hybrid states is obtained by solving the Schrodinger equation for the motion of the quarks in the lowest lying excited potential Given the greatly increased precision with which the E_u potential has been determined, it would be interesting to solve the Schrodinger equation for the motion of the quarks in this potential. Note, however, that the functional dependence on the source separation of the E_u potential remains essentially as previously inferred [1–3] The potential is essentially flat up to a separation of 0 6 fm and eventually rises linearly with the slope of the ground state potential We therefore expect that the main features of the spectrum will remain as previously determined The lowest hybrid state will be close to the minimum of the E_u potential, the hybrids are expected to have broad widths, radial and orbital excitations will be closely spaced in energy, wave functions will be spread out spatially and the clearest experimental signature for the hybrids will remain the exotic quantum numbers of many hybrid states Any details the solution of the Schrodinger equation may add to the spectrum will probably be of limited interest to the experimentalist, because of the assumptions implicit in this calculation the colour SU(3) group of QCD is replaced by SU(2), and the contribution of internal quark loops is ignored

An accurate determination of the lowest lying excited potentials in the context of pure SU(3) gauge theory is probably feasible through the "blocking" technique Although it is generally expected that SU(2) and SU(3) results for potentials are similar, recent studies of the E_u and A_{1u} potentials in lattice SU(3) gauge theory [17] indicate that the latter is the lower lying of the two However, the errors are relatively large and the methods employed tend to underestimate the A_{1u} and to overestimate the E_u potential We intend to carry out the SU(3) calculation along the lines of this work in the near future to investigate this apparent difference between the SU(2) and SU(3) spectra

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